Artificial Gauge Fields for Neutral Atoms

1 Motivation

- Neutral cold atoms as toy model for various physical problems
- Artificial gauge fields: affect neutral atoms like a real magnetic field affects charged particles!
- Possible application: simulating Quantum-Hall regime with cold atoms
- Goal: engineering Hamiltonians for cold neutral atoms which look like Hamiltonians for charged particles moving in a real magnetic field

2 Rotating BEC

- Charged particle: $H = \frac{\left(\vec{p} - q\,\vec{A}(\vec{r},t)\right)^2}{2\,m} + q\,V(\vec{r},t)$
- Rotating neutral atom: $H = \frac{\vec{p}^2}{2\,m} + \frac{1}{2}\,m\,\omega^2\,r^2 - \vec{\Omega}\,\vec{L}$
- Rotation induces a artificial vector potential

$$H = \frac{\vec{p}^2}{2\,m} + \frac{1}{2}\,m\,\omega^2\,r^2 - \vec{\Omega}\,\vec{L} = \frac{\left(\vec{p} - \vec{A}(\vec{r})\right)^2}{2\,m} + \frac{1}{2}\,m\,\left(\omega^2 - \Omega^2\right)\,r^2 \tag{1}$$

- $q = 1 \quad V(\vec{r}) = \frac{1}{2}\,m\,\left(\omega^2 - \Omega^2\right)\,r^2 \quad \vec{A}(\vec{r}) = \Omega\,m \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}$

3 Vortex generation using lasers and real magnetic fields

- BEC of ^{87}Rb, $F = 1$
- Use lasers and magnetic field to change eigenstates
- Use dispersion relation for k_x as effective Hamiltonian: $E\,(k_x) \approx \frac{\hbar^2\,(k_x - k_{min})^2}{2\,m^*}$
- Make k_{min} space-dependent $\rightarrow$ non-zero artificial magnetic field

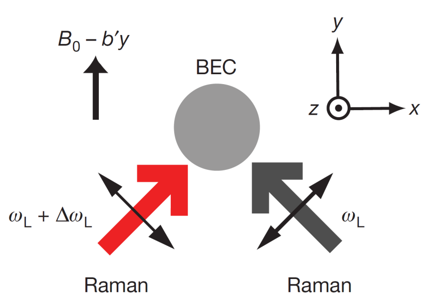

Figure 1: Setup geometry[6]

4 Hofstadter butterfly for cold neutral atoms

- Electrons in a periodic potential → Bloch waves: $\psi_{n\vec{k}}(\vec{r}) = u_{n\vec{k}}(\vec{r})\, e^{i\vec{k}\vec{r}}$
- 2D-square-lattice (x-y-plane) with lattice spacing a
- Bloch energy function: $W\left(\vec{k}\right) = 2\,E_0\,\left(\cos(k_x a) + \cos(k_y a)\right)$
- Harper's equation: $g(m+1) + g(m-1) + 2\cos(2\,\pi\,m\,\alpha - \nu)\,g(m) = \epsilon\,g(m)$

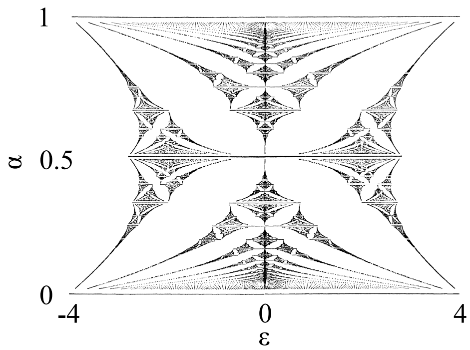

Figure 2: Hofstadter butterfly[4]

Proposal for experimental realisation

- Cold atoms in 2D optical lattice → Description with Bose-Hubbard-model
- Create parameter $\alpha = \frac{q\,\lambda}{4\,\pi}$, tunable with laser (q: wavenuber of coupling lasers, λ: wavelangth of trapping laser)
- Engineering of the Hamiltonian: $H(\alpha) = J\,\Sigma_{m,n}\left(e^{2\,\pi i\,\alpha\,m}\,c^{\dagger}_{n,m}\,c_{n+1,m} + c^{\dagger}_{n,m}\,c_{n,m+1} + h.c.\right)$
- Equivalent to H for electrons moving on lattice with magnetic field $B = \frac{2\pi\alpha}{A\,e}$ with $A = a_x\,a_y$ (area of one lattice cell)

References

[1] *Advanced Quantum Mechanics II.* `http://oer.physics.manchester.ac.uk/`

[2] *Quantum Hall Effect.* `http://homepages.warwick.ac.uk/`

[3] Goerbig, M. O.: Quantum Hall Effects. In: *arXiv:0909.1998v2* (1998)

[4] Hofstadter, D. R.: Energy levels and wave functions of Bloch electrons in rational and irrational magnetic fields. In: *Phys. Rev. B* 14 (1976), Nr. 6, S. 2239–2249

[5] Jaksch, D. ; Zoller, P.: Creation of effective magnetic fields in optical lattices: the Hofstadter butterfly for cold neutral atoms. In: *New J. Phys.* 5 (2003), Nr. 56

[6] Lin, Y.-J. ; Compton, R. L. ; Jiménez-García, K. ; Porto, J. V. ; Spielman, I. B.: Synthetic magnetic fields for ultracold neutral atoms. In: *nature* 462 (2009), S. 628–632

[7] Lin, Y.-J. ; Compton, R. L. ; Perry, A. R. ; Phillips, W. D. ; Porto, J. V. ; Spielman, I. B.: Bose-Einstein Condensate in a Uniform Light-Induced Vector Potential. In: *Phys. Rev. Lett.* 102 (2009), S. 130401