Boson coherent states

Christian Zimmer

Motivation

Identical particles

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Summar

# Quantum field-theory of low dimensional systems

- Coherent states for bosons -

Christian Zimmer

April 22, 2014



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Recap Identica particles  $b^{\dagger}$  and

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- 2 Recapitulation: 2nd quantization
  - Systems of identical particles
  - Creation and annihilation operators
- 3 Coherent states for Bosons
  - Motivation
  - Boson coherent states construction
  - Properties and consequences
- 4 Summary

- ▶ Single particle: express operators in terms of  $\hat{x}$  and  $\hat{p}$ 
  - $\longrightarrow$  natural representation = coordinate representation (eigenfunctions of position operator  $\hat{x}$ )
- ▶ System of identical particles: 2nd quantization, express operators in terms of  $b^{\dagger}$  and b
  - natural representation = coherent state representation (eigenstates of annihilation operator b)
- ► Coherent states provide important representation for path integral formalism of many-particle systems
- Coherent states represent the most classical-like states (e.g.:  $\Delta \hat{x} \ \Delta \hat{p} = \frac{\hbar}{2}, \ \langle \hat{\vec{E}} \rangle \neq 0$ )



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- ► Hilbert space for *N* particles:  $\mathcal{H}_N = \underbrace{\mathcal{H} \otimes \mathcal{H} \otimes ... \otimes \mathcal{H}}_{N \text{ times}}$
- ▶ in nature: only totally symmetric or antisymmetric states are observed  $\Rightarrow \mathcal{H}_N = \mathcal{B}_N \oplus \mathcal{F}_N$
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- ▶ physical bosonic system: **no restriction** on total number of particles  $N = \sum_{i=1}^{\infty} n_{\alpha_i}$  nor on particles  $n_{\alpha_i}$  in state  $|\alpha_i\rangle$ 
  - $\longrightarrow$  Fock space:  $\mathcal{B} \equiv \mathcal{B}_0 \oplus \mathcal{B}_1 \oplus ... = \bigoplus_{n=0}^{\infty} \mathcal{B}_n$
  - ONB for  $\mathcal{B}$ :  $\{|0\rangle, |\alpha_1\rangle, |\alpha_1\alpha_2\rangle, ... |\alpha_1\alpha_2...\alpha_i...\rangle, ...\}$ =  $\{|n_{\alpha}, n_{\alpha_2}, ..., n_{\alpha_i}, ...\rangle\} \equiv \{|n_1n_2, ..., n_i, ...\rangle\}$
  - $\longrightarrow$  construction:  $|\alpha_1\alpha_2...\alpha_N\rangle = \frac{1}{\sqrt{\prod\limits_{\alpha}n_{\alpha}!}}b_1^{\dagger}b_2^{\dagger} \cdot \cdot \cdot \cdot \cdot b_N^{\dagger}|0\rangle$

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- **Hilbert space** for *N* particles:  $\mathcal{H}_N = \mathcal{H} \otimes \mathcal{H} \otimes ... \otimes \mathcal{H}$ N times
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- $b^{\dagger}$  and b provide representation for many-particle states and basis for many-body operators
  - ▶ Definitions:

$$b_i^{\dagger} \mid n_1 n_2 \dots n_i \dots \rangle \equiv \sqrt{n_i + 1} \mid n_1 n_2 \dots n_i + 1 \dots \rangle$$
 $b_i \equiv (b_i^{\dagger})^{\dagger} \Rightarrow b_i \mid n_1 n_2 \dots n_i \dots \rangle = \sqrt{n_i} \mid n_1 n_2 \dots n_i - 1 \dots \rangle$ 
 $\hat{n}_i \equiv b_i^{\dagger} b_i \Rightarrow \hat{n}_i \mid n_1 n_2 \dots n_i \dots \rangle = n_i \mid n_1 n_2 \dots n_i \dots \rangle$ 

- $ightharpoonup b^{\dagger}$  and b operate in Fock space instead of Hilbert space
- ► symmetry property ⇒ commutation relations:

$$[b_i^\dagger,b_i^\dagger]=0=[b_i,b_j]$$
 and  $[b_i,b_i^\dagger]=\delta_{ij}$ 

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- normally: basis of **position** eigenstates  $|\vec{r}\rangle$ 
  - $\longrightarrow$  **permanents** as natural basis of  ${\cal B}$

$$\psi_{\alpha_1...\alpha_N}(\vec{r}_1,...\vec{r}_N) = (\vec{r}_1 ... \vec{r}_N | \alpha_1 ... \alpha_N) = \frac{Per(\psi_{\alpha_i}(\vec{r}_j))}{\sqrt{N! \prod_{\alpha} n_{\alpha}!}}$$

- ightharpoonup now second quantization:  $(\hat{x},\hat{p}) \longrightarrow (b_{\alpha}^{\dagger},b_{\alpha})$ 
  - $\longrightarrow$  basis of eigenstates  $|\Phi\rangle$  of **annihilation operator** b
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  - $\longrightarrow$  is not an ONB but spans the whole Fock space

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#### Why eigenstates of b instead of $b^{\dagger}$ ?

consider general state  $|\Phi\rangle$  of Fock space  $\mathcal{B}$ :

$$\begin{split} |\Phi\rangle &= \sum_{n=0}^{\infty} \sum_{\alpha_{1}...\alpha_{n}} \Phi_{\alpha_{1}...\alpha_{n}} |\alpha_{1} ... \alpha_{n}\rangle \\ &= \sum_{n_{\alpha_{1}},...n_{\alpha_{i}},...} \Phi_{n_{\alpha_{1}}...n_{\alpha_{i}}...} |n_{\alpha_{1}} ... n_{\alpha_{i}} ...\rangle \end{split}$$

- $\begin{array}{lll} \longrightarrow |\Phi'\rangle \equiv b^{\dagger}_{\alpha_j} \, |\Phi\rangle \; \Rightarrow \; n'_{\alpha_j} = n_{\alpha_j} + 1 \; \text{for every component} \\ \Rightarrow \; \left(\text{component of } |\Phi\rangle \; \text{with lowest } N\right) \notin \; |\Phi'\rangle \; \text{anymore} \\ \Rightarrow \; |\Phi'\rangle = b^{\dagger}_{\alpha_j} \, |\Phi\rangle \; \propto \; |\Phi\rangle \; \; \text{is not possible} \\ \Rightarrow \; b^{\dagger}_{\alpha} \; \text{cannot have an eigenstate} \end{array}$
- $\longrightarrow$  but:  $|\Phi''\rangle \equiv b_{\alpha_j} |\Phi\rangle \Rightarrow n''_{\alpha_j} = n_{\alpha_j} 1$  for every component  $\Rightarrow$  since  $n_{\alpha_j} \in \{0, \infty\}, |\Phi''\rangle \propto |\Phi\rangle$  is possible  $\Rightarrow b_{\alpha}$  can have an eigenstate  $|\Phi\rangle$

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assuming  $|\Phi\rangle$  is eigenstate of any  $b_{\alpha} \Rightarrow b_{\alpha_i} |\Phi\rangle = \Phi_{\alpha_i} |\Phi\rangle$ 

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assuming  $|\Phi\rangle$  is eigenstate of any  $b_{lpha} \; \Rightarrow \; b_{lpha_i} \, |\Phi\rangle \; = \; \Phi_{lpha_i} \, |\Phi\rangle$ 

convenient expansion: 
$$|\Phi\rangle = \sum_{n_{\alpha_1},...n_{\alpha_i},...} \Phi_{n_{\alpha_1}...n_{\alpha_i}...} |n_{\alpha_1}...n_{\alpha_i}...\rangle$$

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$$|\Phi\rangle = \sum_{n_{\alpha_1}, \dots n_{\alpha_i}, \dots} \Phi_{n_{\alpha_1} \dots n_{\alpha_i}, \dots} |n_{\alpha_1} \dots n_{\alpha_i} \dots\rangle$$

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$$\Rightarrow \Phi_{n_{\alpha_1} \dots n_{\alpha_i}, \dots} = \frac{\Phi_{\alpha_i}^{n_{\alpha_i}}}{\sqrt{n_{\alpha_i}!}} \Phi_{n_{\alpha_1} \dots 0, \dots} = \frac{\Phi_{\alpha_1}^{n_{\alpha_1}}}{\sqrt{n_{\alpha_1}!}} \dots \frac{\Phi_{\alpha_i}^{n_{\alpha_i}}}{\sqrt{n_{\alpha_i}!}} \dots$$

using 
$$|n_{\alpha_1}...n_{\alpha_i}...\rangle = \frac{b^{\dagger n_{\alpha_1}}_{\alpha_1}}{\sqrt{n_{\alpha_1}!}} \cdots \frac{b^{\dagger n_{\alpha_i}}_{\alpha_i}}{\sqrt{n_{\alpha_i}!}} \cdots |0\rangle$$
:

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#### Construction

assuming  $|\Phi\rangle$  is eigenstate of any  $b_{\alpha} \Rightarrow b_{\alpha_i} |\Phi\rangle = \Phi_{\alpha_i} |\Phi\rangle$ 

$$\begin{array}{ll} \text{convenient expansion:} & |\Phi\rangle = \sum\limits_{n_{\alpha_1}, \dots n_{\alpha_i}, \dots} \Phi_{n_{\alpha_1} \dots n_{\alpha_i} \dots} | \underline{n_{\alpha_1} \dots n_{\alpha_i} \dots} \rangle \\ \Rightarrow & \sqrt{n_{\alpha_i}} \; \Phi_{n_{\alpha_1} \dots n_{\alpha_i}, \dots} = \; \Phi_{\alpha_i} \Phi_{n_{\alpha_1} \dots n_{\alpha_i} - 1 \dots} \\ \Rightarrow & \Phi_{n_{\alpha_1} \dots n_{\alpha_i}, \dots} = \; \frac{\Phi_{\alpha_i}^{n_{\alpha_i}}}{\sqrt{n_{\alpha_i}!}} \; \Phi_{n_{\alpha_1} \dots 0 \dots} = \; \frac{\Phi_{\alpha_1}^{n_{\alpha_1}}}{\sqrt{n_{\alpha_1}!}} \dots \frac{\Phi_{\alpha_i}^{n_{\alpha_i}}}{\sqrt{n_{\alpha_i}!}} \dots \end{array}$$

using 
$$|\mathbf{n}_{\alpha_{1}}...\mathbf{n}_{\alpha_{i}}...\rangle = \frac{b^{\dagger n_{\alpha_{1}}}_{\alpha_{1}!}}{\sqrt{n_{\alpha_{1}}!}} \cdots \frac{b^{\dagger n_{\alpha_{i}}}_{\alpha_{i}}}{\sqrt{n_{\alpha_{i}}!}} \cdots |0\rangle$$
:  

$$\Rightarrow |\Phi\rangle = \sum_{n_{\alpha_{1}},...n_{\alpha_{i}},...} \prod_{i=1}^{\infty} \frac{(\Phi_{\alpha_{i}}b^{\dagger}_{\alpha_{i}})^{n_{\alpha_{i}}}}{n_{\alpha_{i}}!} |0\rangle = \prod_{i=1}^{\infty} \sum_{n_{\alpha_{i}}} \frac{(\Phi_{\alpha_{i}}b^{\dagger}_{\alpha_{i}})^{n_{\alpha_{i}}}}{n_{\alpha_{i}}!} |0\rangle$$

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#### Construction

assuming  $|\Phi\rangle$  is eigenstate of any  $b_{\alpha} \Rightarrow b_{\alpha_i} |\Phi\rangle = \Phi_{\alpha_i} |\Phi\rangle$ 

convenient expansion: 
$$|\Phi\rangle = \sum_{n_{\alpha_1}, \dots n_{\alpha_i}, \dots} \Phi_{n_{\alpha_1} \dots n_{\alpha_i}, \dots} |n_{\alpha_1} \dots n_{\alpha_i} \dots\rangle$$

$$\Rightarrow \sqrt{n_{\alpha_i}} \Phi_{n_{\alpha_1} \dots n_{\alpha_i}, \dots} = \Phi_{\alpha_i} \Phi_{n_{\alpha_1} \dots n_{\alpha_i} - 1, \dots}$$

$$\Rightarrow \Phi_{n_{\alpha_1} \dots n_{\alpha_i}, \dots} = \frac{\Phi_{\alpha_i}^{n_{\alpha_i}}}{\sqrt{n_{\alpha_i}!}} \Phi_{n_{\alpha_1} \dots 0, \dots} = \frac{\Phi_{\alpha_1}^{n_{\alpha_1}}}{\sqrt{n_{\alpha_1}!}} \dots \frac{\Phi_{\alpha_i}^{n_{\alpha_i}}}{\sqrt{n_{\alpha_i}!}} \dots$$

using 
$$|n_{\alpha_{1}}...n_{\alpha_{i}}...\rangle = \frac{b^{\lceil n_{\alpha_{1}}}_{\alpha_{1}}}{\sqrt{n_{\alpha_{1}}!}} \cdots \frac{b^{\lceil n_{\alpha_{i}}}_{\alpha_{i}}}{\sqrt{n_{\alpha_{i}}!}} \cdots |0\rangle$$
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$$\Rightarrow |\Phi\rangle = \sum_{n_{\alpha_{1}},...n_{\alpha_{i}},...} \prod_{i=1}^{\infty} \frac{(\Phi_{\alpha_{i}}b^{\dagger}_{\alpha_{i}})^{n_{\alpha_{i}}}}{n_{\alpha_{i}}!} |0\rangle = \prod_{i=1}^{\infty} \sum_{n_{\alpha_{i}}} \frac{(\Phi_{\alpha_{i}}b^{\dagger}_{\alpha_{i}})^{n_{\alpha_{i}}}}{n_{\alpha_{i}}!} |0\rangle$$

$$= \prod_{\alpha} \exp\left(\Phi_{\alpha}b^{\dagger}_{\alpha}\right) |0\rangle = \left[\exp\left(\sum_{\alpha} \Phi_{\alpha}b^{\dagger}_{\alpha}\right) |0\rangle\right]$$

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## Properties and consequences

Useful relations

$$b_{lpha} \ket{\Phi} = \ket{\Phi_{lpha}} \ket{\Phi} \qquad \Rightarrow \qquad \ket{\Phi} = \exp \left( \sum_{lpha} \Phi_{lpha} b_{lpha}^{\dagger} \right) \ket{0}$$

⇒ adjoint expression:

$$\langle \Phi | b_{\alpha}^{\dagger} = \langle \Phi | \Phi_{\alpha}^{*} \Rightarrow \langle \Phi | = \langle 0 | \exp \left( \sum_{\alpha} \Phi_{\alpha}^{*} b_{\alpha} \right) \right)$$

action of creation operators on coherent states:

$$b_{\alpha}^{\dagger} |\Phi\rangle = b_{\alpha}^{\dagger} \exp \left( \sum_{\alpha'} \Phi_{\alpha'} b_{\alpha'}^{\dagger} \right) |0\rangle = \frac{\partial}{\partial \Phi_{\alpha}} |\Phi\rangle$$

 $\Rightarrow$  adjoint expression

$$\langle \Phi | \, b_{\alpha} = \langle \Phi | \exp \left( \sum_{\alpha'} \Phi_{\alpha'}^* b_{\alpha'} \right) b_{\alpha} = \frac{\partial}{\partial \Phi_{\alpha}^*} | \Phi \rangle$$

## Properties and consequences

Useful relations

$$b_{lpha} \left| \Phi \right> = \left| \Phi_{lpha} \left| \Phi \right> \right| \Rightarrow \left| \Phi \right> = \exp \left( \sum_{lpha} \Phi_{lpha} b_{lpha}^{\dagger} \right) \left| 0 \right>$$

 $\Rightarrow$  adjoint expression:

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## Properties and consequences

Useful relations

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$$\langle \Phi | b_{\alpha} = \langle \Phi | \exp \left( \sum_{\alpha'} \Phi_{\alpha'}^* b_{\alpha'} \right) b_{\alpha} = \frac{\partial}{\partial \Phi_{\alpha}^*} | \Phi \rangle$$

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## Properties and consequences

Useful relations

$$b_{lpha} \ket{\Phi} = \ket{\Phi_{lpha}} \ket{\Phi} \qquad \Rightarrow \qquad \ket{\Phi} = \exp \left( \sum_{lpha} \Phi_{lpha} b_{lpha}^{\dagger} \right) \ket{0}$$

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action of creation operators on coherent states:

$$|b_{\alpha}^{\dagger}|\Phi\rangle = |b_{\alpha}^{\dagger}|\Phi\rangle = |b_{\alpha}^{\dagger}|\Phi\rangle = |\Phi\rangle = |\Phi\rangle$$

 $\Rightarrow$  adjoint expression:

$$ra{race{\Phi}} b_lpha = race{\Phi} \exp\left(\sum_{lpha'} \Phi_{lpha'}^* b_{lpha'}\right) b_lpha = \left. rac{\partial}{\partial \Phi_lpha^*} \right. \ket{\Phi}$$

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## Properties and consequences Overlap

$$\begin{split} \langle \Phi | \Phi' \rangle \; &= \; \sum_{\substack{n_{\alpha_{1}}, \dots n_{\alpha_{i}}, \dots \\ n'_{\alpha_{1}}, \dots n'_{\alpha_{i}}, \dots}} \frac{\Phi^{*n_{\alpha_{1}}}_{\alpha_{1}}}{\sqrt{n_{\alpha_{1}}!}} \cdots \frac{\Phi^{*n_{\alpha_{i}}}_{\alpha_{i}}}{\sqrt{n_{\alpha_{i}}!}} \cdots \frac{\Phi'^{n'_{\alpha_{1}}}_{\alpha_{1}}}{\sqrt{n'_{\alpha_{1}}!}} \cdots \frac{\Phi'^{n'_{\alpha_{i}}}_{\alpha_{i}}}{\sqrt{n'_{\alpha_{i}}!}} \cdots \\ & \times \underbrace{\langle n_{\alpha_{1}} \dots n_{\alpha_{i}} \dots | n'_{\alpha_{1}} \dots n'_{\alpha_{i}} \dots \rangle}_{\prod\limits_{i=1}^{\infty} \delta_{n_{\alpha_{i}}} n'_{\alpha_{i}}} \end{split}$$

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## Properties and consequences Overlap

$$\begin{split} \langle \Phi | \Phi' \rangle \; &= \; \sum_{\substack{n_{\alpha_{1}}, \dots n_{\alpha_{i}}, \dots \\ n'_{\alpha_{1}}, \dots n'_{\alpha_{i}}, \dots}} \frac{\Phi^{*n_{\alpha_{1}}}_{\alpha_{1}}}{\sqrt{n_{\alpha_{1}}!}} \cdots \frac{\Phi^{*n_{\alpha_{i}}}_{\alpha_{i}}}{\sqrt{n_{\alpha_{i}}!}} \cdots \frac{\Phi'^{n'_{\alpha_{1}}}_{\alpha_{1}}}{\sqrt{n'_{\alpha_{1}}!}} \cdots \frac{\Phi'^{n'_{\alpha_{i}}}_{\alpha_{i}}}{\sqrt{n'_{\alpha_{i}}!}} \cdots \\ & \times \underbrace{\langle n_{\alpha_{1}} \dots n_{\alpha_{i}} \dots | n'_{\alpha_{1}} \dots n'_{\alpha_{i}} \dots \rangle}_{\prod\limits_{i=1}^{\infty} \delta_{n_{\alpha_{i}}} n'_{\alpha_{i}}} \\ &= \; \sum_{n_{\alpha_{1}} \dots n_{\alpha_{i}} \dots \dots} \underbrace{\frac{\left(\Phi^{*}_{\alpha_{1}} \Phi'_{\alpha_{1}}\right)^{n_{\alpha_{1}}}}{n_{\alpha_{1}}!} \cdots \underbrace{\left(\Phi^{*}_{\alpha_{i}} \Phi'_{\alpha_{i}}\right)^{n_{\alpha_{i}}}}_{n_{\alpha_{i}}!} \cdots }_{n_{\alpha_{i}}!} \cdots \end{split}$$

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# Properties and consequences Overlap

$$\begin{split} \langle \Phi | \Phi' \rangle \; &= \; \sum_{\substack{n_{\alpha_{1}, \dots, n_{\alpha_{i}}, \dots \\ n'_{\alpha_{1}}, \dots, n'_{\alpha_{i}}, \dots \\ n'_{\alpha_{1}}, \dots, n'_{\alpha_{i}}, \dots \\ }} \frac{\Phi^{* \frac{n_{\alpha_{1}}}{\alpha_{1}}}}{\sqrt{n_{\alpha_{1}}!}} \cdots \frac{\Phi^{* \frac{n_{\alpha_{i}}}{\alpha_{i}}}}{\sqrt{n_{\alpha_{i}}!}} \cdots \frac{\Phi'^{n'_{\alpha_{1}}}{\alpha_{1}}}{\sqrt{n'_{\alpha_{1}}!}} \cdots \frac{\Phi'^{n'_{\alpha_{i}}}{\alpha_{i}}}{\sqrt{n'_{\alpha_{i}}!}} \cdots \\ & \times \underbrace{\langle n_{\alpha_{1}, \dots, n_{\alpha_{i}}, \dots} | n'_{\alpha_{1}, \dots} n'_{\alpha_{i}}, \dots \rangle}_{\prod\limits_{i=1}^{\infty} \delta_{n_{\alpha_{i}} n'_{\alpha_{i}}}} \\ & = \; \sum_{n_{\alpha_{1}, \dots, n_{\alpha_{i}}, \dots}} \frac{\left(\Phi^{*}_{\alpha_{1}} \Phi'_{\alpha_{1}}\right)^{n_{\alpha_{1}}}}{n_{\alpha_{1}}!} \cdots \frac{\left(\Phi^{*}_{\alpha_{i}} \Phi'_{\alpha_{i}}\right)^{n_{\alpha_{i}}}}{n_{\alpha_{i}}!} \cdots \\ & = \; \prod \sum\limits_{n=1}^{\infty} \frac{\left(\Phi^{*}_{\alpha} \Phi'_{\alpha}\right)^{n_{\alpha}}}{n_{\alpha_{i}}!} \quad = \; \exp\left(\sum \Phi^{*}_{\alpha} \Phi'_{\alpha}\right) \end{split}$$

## Properties and consequences

Overlap

$$\begin{split} \langle \Phi | \Phi' \rangle \; &= \; \sum_{\substack{n_{\alpha_{1}, \dots, n_{\alpha_{i}}, \dots \\ n'_{\alpha_{1}}, \dots, n'_{\alpha_{i}}, \dots \\ n'_{\alpha_{1}}, \dots, n'_{\alpha_{i}}, \dots \\ }} \frac{\Phi^{*n_{\alpha_{1}}}_{\alpha_{1}}}{\sqrt{n_{\alpha_{1}}!}} \cdots \frac{\Phi^{*n_{\alpha_{i}}}_{\alpha_{i}}}{\sqrt{n_{\alpha_{i}}!}} \cdots \frac{\Phi'^{n'_{\alpha_{1}}}_{\alpha_{1}}}{\sqrt{n'_{\alpha_{1}}!}} \cdots \frac{\Phi'^{n'_{\alpha_{i}}}_{\alpha_{i}}}{\sqrt{n'_{\alpha_{i}}!}} \cdots \\ & \times \underbrace{\langle n_{\alpha_{1}} \dots n_{\alpha_{i}} \dots | n'_{\alpha_{1}} \dots n'_{\alpha_{i}} \dots \rangle}_{\prod\limits_{i=1}^{\infty} \delta_{n_{\alpha_{i}} n'_{\alpha_{i}}}} \\ &= \; \sum_{n_{\alpha_{1}}, \dots, n_{\alpha_{i}}, \dots} \frac{\left(\Phi^{*}_{\alpha_{1}} \Phi'_{\alpha_{1}}\right)^{n_{\alpha_{1}}}}{n_{\alpha_{1}}!} \cdots \frac{\left(\Phi^{*}_{\alpha_{i}} \Phi'_{\alpha_{i}}\right)^{n_{\alpha_{i}}}}{n_{\alpha_{i}}!} \cdots \\ &= \; \prod \sum\limits_{n=1}^{\infty} \frac{\left(\Phi^{*}_{\alpha} \Phi'_{\alpha}\right)^{n_{\alpha}}}{n_{\alpha_{i}}!} \; = \; \exp\left(\sum\limits_{i=1}^{\infty} \Phi^{*}_{\alpha} \Phi'_{\alpha}\right) \end{split}$$

⇒ coherent states are neither orthogonal nor normalized



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### Properties and consequences

Overcompleteness

### closure relation:

$$\prod_{\alpha} \int_{\mathbb{C}} \frac{\mathsf{d} \Phi_{\alpha}^{*} \mathsf{d} \Phi_{\alpha}}{2 \imath \pi} \exp \left( - \sum_{\alpha} \Phi_{\alpha}^{*} \Phi_{\alpha} \right) |\Phi\rangle \langle \Phi| = \mathbb{1}_{\mathcal{B}}$$

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### Properties and consequences

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#### closure relation:

$$\prod_{\alpha} \int\limits_{\mathbb{C}} \frac{\mathsf{d} \Phi_{\alpha}^* \mathsf{d} \Phi_{\alpha}}{2 \imath \pi} \exp \left( - \sum_{\alpha} \Phi_{\alpha}^* \Phi_{\alpha} \right) |\Phi\rangle \langle \Phi| \ = \ \mathbb{1}_{\mathcal{B}}$$

#### Proof:

1) Start with state  $|n\rangle \in \mathcal{B}$  representing n particles in state  $|\alpha\rangle$ 

$$\int\limits_{\mathbb{C}} \frac{\mathsf{d}\Phi^*\mathsf{d}\Phi}{2\imath\pi} e^{-\Phi^*\Phi} \left|\Phi\right\rangle \left\langle\Phi\right| \; = \; \int\limits_{\mathbb{C}} \frac{\mathsf{d}\Phi^*\mathsf{d}\Phi}{2\imath\pi} e^{-\Phi^*\Phi} \sum_{n,m} \frac{\Phi^n}{\sqrt{n!}} \left|n\right\rangle \left\langle m\right| \frac{{\Phi^*}^m}{\sqrt{m!}}$$

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#### closure relation:

$$\prod_{\alpha} \int_{\mathbb{C}} \frac{\mathsf{d} \Phi_{\alpha}^{*} \mathsf{d} \Phi_{\alpha}}{2 \imath \pi} \exp \left( - \sum_{\alpha} \Phi_{\alpha}^{*} \Phi_{\alpha} \right) |\Phi\rangle \langle \Phi| \ = \ \mathbb{1}_{\mathcal{B}}$$

$$\int\limits_{\mathbb{C}} \frac{\mathsf{d}\Phi^*\mathsf{d}\Phi}{2\imath\pi} \, e^{-\Phi^*\Phi} \, |\Phi\rangle \, \langle\Phi| \; = \; \int\limits_{\mathbb{C}} \frac{\mathsf{d}\Phi^*\mathsf{d}\Phi}{2\imath\pi} \, e^{-\Phi^*\Phi} \sum_{n,m} \frac{\Phi^n}{\sqrt{n!}} \, |n\rangle \, \langle m| \, \frac{\Phi^{*m}}{\sqrt{m!}}$$

### Properties and consequences

Overcompleteness

#### closure relation:

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$$\begin{split} \int\limits_{\mathbb{C}} \frac{\mathrm{d}\Phi^*\mathrm{d}\Phi}{2\imath\pi} e^{-\Phi^*\Phi} \left|\Phi\right\rangle \left\langle\Phi\right| &= \int\limits_{\mathbb{C}} \frac{\mathrm{d}\Phi^*\mathrm{d}\Phi}{2\imath\pi} e^{-\Phi^*\Phi} \sum_{n,m} \frac{\Phi^n}{\sqrt{n!}} \left|n\right\rangle \left\langle m\right| \frac{{\Phi^*}^m}{\sqrt{m!}} \\ \stackrel{\Phi=\rho e^{\imath\theta}}{=} \int\int \frac{\rho \ \mathrm{d}\rho \ \mathrm{d}\theta}{\pi} e^{-\rho^2} \sum_{n,m} \frac{(\rho e^{\imath\theta})^n}{\sqrt{n!}} \frac{(\rho e^{-\imath\theta})^m}{\sqrt{m!}} \ \left|n\right\rangle \left\langle m\right| \end{split}$$

### Properties and consequences

Overcompleteness

#### closure relation:

$$\prod_{\alpha} \int_{\mathbb{C}} \frac{\mathsf{d} \Phi_{\alpha}^* \mathsf{d} \Phi_{\alpha}}{2 \imath \pi} \exp \left( - \sum_{\alpha} \Phi_{\alpha}^* \Phi_{\alpha} \right) |\Phi\rangle \langle \Phi| \ = \ \mathbb{1}_{\mathcal{B}}$$

$$\begin{split} \int \frac{\mathrm{d}\Phi^*\mathrm{d}\Phi}{2\imath\pi} \, e^{-\Phi^*\Phi} \, |\Phi\rangle \, \langle\Phi| &= \int \limits_{\mathbb{C}} \frac{\mathrm{d}\Phi^*\mathrm{d}\Phi}{2\imath\pi} \, e^{-\Phi^*\Phi} \sum_{n,m} \frac{\Phi^n}{\sqrt{n!}} \, |n\rangle \, \langle m| \, \frac{\Phi^{*m}}{\sqrt{m!}} \\ \stackrel{\Phi=\rho e^{\imath\theta}}{=} \int \int \frac{\rho}{\pi} \, \mathrm{d}\rho \, \, \mathrm{d}\theta \, e^{-\rho^2} \sum_{n,m} \frac{(\rho e^{\imath\theta})^n}{\sqrt{n!}} \, \frac{(\rho e^{-\imath\theta})^m}{\sqrt{m!}} \, |n\rangle \, \langle m| \\ = \int \frac{\rho}{\pi} \mathrm{d}\rho \, \, e^{-\rho^2} \sum_{n,m} \frac{\rho^{n+m}}{\sqrt{n!m!}} \, \int \mathrm{d}\theta \, \, e^{\imath(n-m)\theta} \, |n\rangle \, \langle m| \end{split}$$

### Properties and consequences

Overcompleteness

#### closure relation:

$$\prod_{\alpha} \int_{\mathbb{C}} \frac{\mathsf{d} \Phi_{\alpha}^* \mathsf{d} \Phi_{\alpha}}{2 \imath \pi} \exp \left( - \sum_{\alpha} \Phi_{\alpha}^* \Phi_{\alpha} \right) |\Phi\rangle \langle \Phi| \ = \ \mathbb{1}_{\mathcal{B}}$$

$$\begin{split} \int \frac{\mathrm{d}\Phi^*\mathrm{d}\Phi}{2\imath\pi} \, e^{-\Phi^*\Phi} \, |\Phi\rangle \, \langle\Phi| &= \int \limits_{\mathbb{C}} \frac{\mathrm{d}\Phi^*\mathrm{d}\Phi}{2\imath\pi} \, e^{-\Phi^*\Phi} \sum_{n,m} \frac{\Phi^n}{\sqrt{n!}} \, |n\rangle \, \langle m| \, \frac{\Phi^{*m}}{\sqrt{m!}} \\ \stackrel{\Phi=\rho e^{\imath\theta}}{=} \int \int \frac{\rho}{\pi} \, \mathrm{d}\rho \, \, \mathrm{d}\theta \, e^{-\rho^2} \sum_{n,m} \frac{(\rho e^{\imath\theta})^n}{\sqrt{n!}} \frac{(\rho e^{-\imath\theta})^m}{\sqrt{m!}} \, |n\rangle \, \langle m| \\ &= \int \frac{\rho}{\pi} \mathrm{d}\rho \, \, e^{-\rho^2} \sum_{n,m} \frac{\rho^{n+m}}{\sqrt{n!m!}} \, \underbrace{\int \mathrm{d}\theta \, \, e^{\imath(n-m)\theta}}_{2\pi\delta_{nm}} |n\rangle \, \langle m| \end{split}$$

### Properties and consequences

Overcompleteness

#### closure relation:

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$$\begin{split} &\int_{\mathbb{C}} \frac{\mathrm{d}\Phi^* \mathrm{d}\Phi}{2\imath \pi} \, e^{-\Phi^*\Phi} \, |\Phi\rangle \, \langle\Phi| \; = \; \int_{\mathbb{C}} \frac{\mathrm{d}\Phi^* \mathrm{d}\Phi}{2\imath \pi} \, e^{-\Phi^*\Phi} \sum_{n,m} \frac{\Phi^n}{\sqrt{n!}} \, |n\rangle \, \langle m| \, \frac{\Phi^{*m}}{\sqrt{m!}} \\ &\stackrel{\Phi=\rho e^{\imath\theta}}{=} \; \int \int \frac{\rho}{\pi} \, \mathrm{d}\rho \, \, \mathrm{d}\theta \, e^{-\rho^2} \sum_{n,m} \frac{(\rho e^{\imath\theta})^n}{\sqrt{n!}} \frac{(\rho e^{-\imath\theta})^m}{\sqrt{m!}} \, |n\rangle \, \langle m| \\ &= \; \int \frac{\rho}{\pi} \mathrm{d}\rho \, \, e^{-\rho^2} \sum_{n,m} \frac{\rho^{n+m}}{\sqrt{n!m!}} \underbrace{\int \mathrm{d}\theta \, \, e^{\imath(n-m)\theta}}_{2\pi\delta_{nm}} |n\rangle \, \langle m| \\ &= \; \sum \frac{2}{n!} \; \int \mathrm{d}\rho \, \, e^{-\rho^2} \rho^{2n+1} \, |n\rangle \, \langle n| \end{split}$$

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#### closure relation:

$$\prod_{\alpha} \int\limits_{\mathbb{C}} \frac{\mathsf{d} \Phi_{\alpha}^* \mathsf{d} \Phi_{\alpha}}{2 \imath \pi} \exp \left( - \sum_{\alpha} \Phi_{\alpha}^* \Phi_{\alpha} \right) |\Phi\rangle \langle \Phi| \ = \ \mathbb{1}_{\mathcal{B}}$$

$$\int_{\mathbb{C}} \frac{d\Phi^* d\Phi}{2\imath \pi} e^{-\Phi^*\Phi} |\Phi\rangle \langle \Phi| = \int_{\mathbb{C}} \frac{d\Phi^* d\Phi}{2\imath \pi} e^{-\Phi^*\Phi} \sum_{n,m} \frac{\Phi^n}{\sqrt{n!}} |n\rangle \langle m| \frac{\Phi^{*m}}{\sqrt{m!}}$$

$$\Phi = \frac{\rho}{e^{\imath\theta}} \int_{\mathbb{C}} \frac{\rho d\rho d\theta}{\pi} e^{-\rho^2} \sum_{n,m} \frac{(\rho e^{\imath\theta})^n}{\sqrt{n!}} \frac{(\rho e^{-\imath\theta})^m}{\sqrt{m!}} |n\rangle \langle m|$$

$$= \int_{\mathbb{C}} \frac{\rho}{\pi} d\rho e^{-\rho^2} \sum_{n,m} \frac{\rho^{n+m}}{\sqrt{n!m!}} \underbrace{\int_{2\pi\delta_{nm}} d\theta e^{\imath(n-m)\theta} |n\rangle \langle m|}_{2\pi\delta_{nm}}$$

$$= \sum_{n} \frac{2}{n!} \underbrace{\int_{\mathbb{C}} d\rho e^{-\rho^2} \rho^{2n+1}}_{\mathbb{C}} |n\rangle \langle n|$$

$$= \frac{1}{2} \Gamma(n+1) = \frac{1}{2} \frac{n!}{n!} \text{ for } n \in \mathbb{N}$$

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#### closure relation:

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$$= \sum_{n} \frac{2}{n!} \underbrace{\int_{\mathbb{C}} d\rho e^{-\rho^2} \rho^{2n+1}}_{=\frac{1}{2}n! \text{ for } n \in \mathbb{N}} |n\rangle \langle n| = \sum_{n} |n\rangle \langle n| = \mathbb{1}_{\mathcal{B}}$$

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### Properties and consequences

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#### closure relation:

$$\prod_{\alpha} \int_{\mathbb{C}} \frac{\mathsf{d} \Phi_{\alpha}^{*} \mathsf{d} \Phi_{\alpha}}{2 \imath \pi} \exp \left( - \sum_{\alpha} \Phi_{\alpha}^{*} \Phi_{\alpha} \right) |\Phi\rangle \langle \Phi| = \mathbb{1}_{\mathcal{B}}$$

#### Proof:

1) Start with state  $|n\rangle \in \mathcal{B}$  representing n particles in state  $|\alpha\rangle$ 

$$\int\limits_{\mathbb{C}} \frac{\mathrm{d}\Phi^*\mathrm{d}\Phi}{2\imath\pi} e^{-\Phi^*\Phi} \left|\Phi\right\rangle \left\langle\Phi\right| \; = \; \sum_{n} \left|n\right\rangle \left\langle n\right| \; = \; \mathbb{1}_{\mathcal{B}}$$

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Overcompleteness

#### closure relation:

$$\prod_{\alpha} \int\limits_{\mathbb{C}} \frac{\mathsf{d} \Phi_{\alpha}^* \mathsf{d} \Phi_{\alpha}}{2 \imath \pi} \exp \left( - \sum_{\alpha} \Phi_{\alpha}^* \Phi_{\alpha} \right) |\Phi\rangle \langle \Phi| \ = \ \mathbb{1}_{\mathcal{B}}$$

#### Proof:

1) Start with state  $|n\rangle \in \mathcal{B}$  representing n particles in state  $|\alpha\rangle$ 

$$\int\limits_{\mathbb{C}} \frac{\mathrm{d}\Phi^*\mathrm{d}\Phi}{2\imath\pi} e^{-\Phi^*\Phi} \left|\Phi\right\rangle \left\langle\Phi\right| \; = \; \sum_{n} \left|n\right\rangle \left\langle n\right| \; = \; \mathbb{1}_{\mathcal{B}}$$

2) now: set of states  $\{|\alpha_k\rangle\} \Rightarrow |n\rangle \rightarrow |n_{\alpha_1} \dots n_{\alpha_i} \dots\rangle$ 

$$\prod_{\alpha} \int\limits_{\Omega} \frac{\mathsf{d} \Phi_{\alpha}^{*} \mathsf{d} \Phi_{\alpha}}{2 \imath \pi} \mathsf{e}^{-\sum_{\alpha} \Phi_{\alpha}^{*} \Phi_{\alpha}} \left| \Phi \right\rangle \left\langle \Phi \right|$$

Properties

### Properties and consequences

Overcompleteness

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### Properties and consequences

coherent state representation vs. coordinate representation

• representation of arbitrary state  $|\psi\rangle$ :

$$|\psi\rangle \ = \ 1\!\!1 \cdot |\psi\rangle \ = \ \begin{cases} \prod\limits_{\alpha} \int\limits_{\mathbb{C}} \frac{\mathrm{d}\Phi_{\alpha}^* \mathrm{d}\Phi_{\alpha}}{2\imath\pi} \exp\left(-\sum\limits_{\alpha} \Phi_{\alpha}^* \Phi_{\alpha}\right) \left\langle \Phi |\psi \right\rangle |\Phi\rangle \\ \int\limits_{\mathbb{C}} \mathrm{d}\mathbf{r}^3 \left\langle \vec{r} |\psi \right\rangle |\vec{r} \rangle \end{cases}$$

with  $\langle \Phi | \psi \rangle \equiv \psi(\Phi^*)$ : coherent state representation of  $|\psi\rangle$  cf.:  $\langle \vec{r} | \psi \rangle \equiv \psi(\vec{r})$ : coordinate representation of  $|\psi\rangle$ 

Christian Zimmer

Motivation

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▶ representation of  $b_{\alpha}^{\dagger}$ ,  $b_{\alpha}$  and  $\hat{\mathbf{x}}$ ,  $\hat{\mathbf{p}}$  respectively:

$$\langle \Phi | b_{\alpha}^{\dagger} | \psi \rangle \ = \ \Phi_{\alpha}^{*} \langle \Phi | \psi \rangle \qquad = \quad \Phi_{\alpha}^{*} \ \psi(\Phi^{*}) \quad \Rightarrow \quad b_{\alpha}^{\dagger} \stackrel{\triangle}{=} \Phi_{\alpha}^{*}$$

$$\langle \Phi | b_{\alpha} | \psi \rangle \ = \ \frac{\partial}{\partial \Phi_{\alpha}^{*}} \langle \Phi | \psi \rangle \qquad = \quad \frac{\partial}{\partial \Phi_{\alpha}^{*}} \psi(\Phi^{*}) \quad \Rightarrow \quad b_{\alpha} \stackrel{\triangle}{=} \frac{\partial}{\partial \Phi_{\alpha}^{*}}$$

cf.:  $\hat{\mathbf{x}} \stackrel{\triangle}{=} \vec{\mathbf{r}}$  and  $\hat{\mathbf{p}} \stackrel{\triangle}{=} -\imath \hbar \frac{\partial}{\partial \vec{\mathbf{r}}}$  (in coordinate representation)

Zimmer

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### Properties and consequences

Matrix elements

coherent state representation:

#### Zimmer

Properties and consequences

Matrix elements

coherent state representation:

$$egin{array}{lll} \langle \Phi | {\cal A}(b_lpha^\dagger,b_lpha) | \Phi' 
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Summan

### Properties and consequences

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simple form of matrix elements of normal-ordered operators  $A(b_{\alpha}^{\dagger},b_{\alpha})$  (normal order: all  $b_{\alpha}^{\dagger}$  to the left of all  $b_{\alpha}$ ):

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Summan

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Construction
Construction
Properties

Summar

# Properties and consequences Poisson distribution

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▶ prob(
$$n_1...n_i...$$
) =  $|\langle n_1...n_i...|\Phi\rangle|^2$   
=  $|\langle n_1...n_i...|\sum_{m_1,...m_i...}\prod_i \frac{\Phi_i^{m_i}}{\sqrt{m_i!}}|m_1...m_i...\rangle|^2$ 

Properties Summary

# Properties and consequences Poisson distribution

 $\hookrightarrow$  independent **Poisson distribution** for each  $n_i$  with mean value  $|\Phi_i|^2$ 

## Properties and consequences

Poisson distribution

- $\hookrightarrow$  independent **Poisson distribution** for each  $n_i$  with mean value  $|\Phi_i|^2$
- ▶ average value of the total number of particles:

$$\overline{N} = \frac{\langle \Phi | \hat{N} | \Phi \rangle}{\langle \Phi | \Phi \rangle} = \frac{\sum_{\alpha} \langle \Phi | b_{\alpha}^{\dagger} b_{\alpha} | \Phi \rangle}{\langle \Phi | \Phi \rangle} = \frac{\sum_{\alpha} \Phi_{\alpha}^{*} \langle \Phi | \Phi \rangle \Phi_{\alpha}}{\langle \Phi | \Phi \rangle} = \sum_{\alpha} |\Phi_{\alpha}|^{2}$$

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▶ variance: 
$$\sigma^2 = \langle \hat{N}^2 \rangle - \langle \hat{N} \rangle^2 = \frac{\langle \Phi | \hat{N}^2 | \Phi \rangle}{\langle \Phi | \Phi \rangle} - \left( \frac{\langle \Phi | \hat{N} | \Phi \rangle}{\langle \Phi | \Phi \rangle} \right)^2 \stackrel{...}{=} \overline{N}$$

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- relative width:  $\frac{\sigma}{\overline{N}} = \frac{1}{\sqrt{\overline{N}}} \xrightarrow{\overline{N} \to \infty} 0$ 
  - $\hookrightarrow$  in TD-limit: coherent states **sharply peaked** around *N*

Motivation Construction

Summary

- 2nd quantization:  $(\hat{x}, \hat{p}) \longrightarrow (b_{\alpha}^{\dagger}, b_{\alpha})$
- ▶ being eigenstates of  $b_{\alpha}$ , coherent states  $|\Phi\rangle$  provide a useful representation for states  $\in$  Fock space  $\mathcal{B}$

- neither orthogonal nor normalized, provide overcomplete basis of Fock space
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