Field-theory for the quantum Heisenberg antiferromagnet in one dimension

Seminar: Quantum field-theory on low dimensional systems

Marco Ströbel

24th june 2014

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- why?

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- different approaches calculate the dispersion-relation e.g. by 2nd quantisation or Bethe-ansatz
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Motivation: experimental measurement

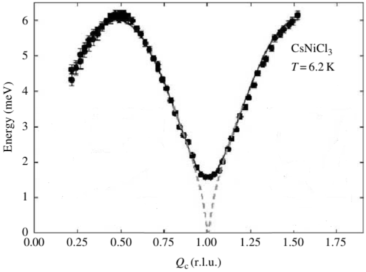

fig.: Neutron scattering for $S = \frac{1}{2}$ and $S = 1$

M. Kenzelmann, R. A. Cowley, W. J. L. Buyers, Z. Tun, R. Coldea and M. Enderle; The properties of Haldane excitations and multi-particle states in the antiferromagnetic spin-1 chain compound CsNiCl3, November 23, 2013

- Measurement of the dispersion relation by neutron scattering
- dashed line: $s = \frac{1}{2} \to$ massless Dirac-particle
- pointed line: $s = 1 \to$ spontaneous mass generation

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Heisenberg model

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- Spin is a quantum mechanical observable $\vec{S}$

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Path-integral-formalism

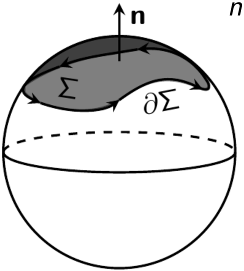
From the talk of Jan Lotze:

- Partition-function:

$$Z = \int D\vec{n}\delta(n^2 - 1)\exp\left(-S[\vec{n}]\right),$$

$$= \int D\vec{n}\delta(n^2 - 1)\exp\left[-\int_0^\beta d\tau\left(\langle\vec{n}|\frac{\partial}{\partial\tau}|\vec{n}\rangle + \langle\vec{n}|\mathcal{H}|\vec{n}\rangle\right)\right]$$

- Coherent states :



$\vec{n}$ is a vector on the unit sphere

$$|\vec{n}\rangle = e^{-i\theta\vec{m}\cdot\vec{S}}|s, -s\rangle,$$

$$\langle\vec{n}|\vec{S}|\vec{n}\rangle = -s\vec{n},\ \ \vec{n}^2 = 1$$

Path-integral-formalism

- kinetic term

$$\int_0^\beta d\tau \langle \vec{n} | \frac{\partial}{\partial \tau} | \vec{n} \rangle = -is \underbrace{\int_0^\beta d\tau [1 - \cos\theta(\tau)] \dot{\varphi}(\tau)}_{=\Omega}$$

- where we assume $\rightarrow \int_{\partial\Sigma} \vec{A} d\vec{n} = \int_\Sigma (\vec{\nabla} \times \vec{A}) \cdot \vec{n} df = \Omega$, where $\vec{A}$ is a vector potential on the unit sphere with $\vec{\nabla} \times \vec{A} = \vec{n}$
- use periodic boundary conditions
- $\partial\Sigma$ is a line integral enclosing the surface on the sphere

Path-integral-formalism

$$Z = \int D\vec{n} \exp\left(\underbrace{-is \sum_j \int_0^\beta d\tau (\vec{A}(\vec{n}_j) \cdot \partial_\tau \vec{n}_j)}_{\text{Berry phase } S_B} - \underbrace{\int_0^\beta d\tau \langle \vec{n} | H | \vec{n} \rangle}_{\text{interaction term } S_{int}} \right)$$

- Heisenberg Hamiltonian:

$$\langle \vec{n} | H | \vec{n} \rangle = J \sum_j \langle \vec{n} | \vec{S}_j \cdot \vec{S}_{j+1} | \vec{n} \rangle = J s^2 \sum_j \vec{n}(j) \cdot \vec{n}(j+1)$$

for nearest neighbor interaction

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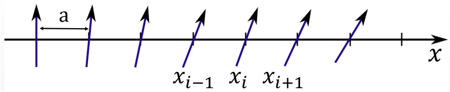
Spin-systems



- $\vec{n}_j = (-1)\vec{n}_{j+1}$and with „$a$“as lattice spacing
- but the spins are staggered
  divide $\vec{n}$ into a slowly varying part $\vec{m}$ and a small but fast fluctuation part $\vec{l}$

$$\vec{n}_j = (-1)^j \sqrt{1 - a^2 l_j^2}\,\vec{m}_j + a\vec{l}_j$$

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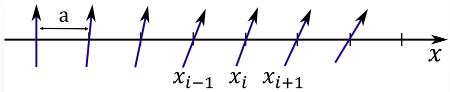

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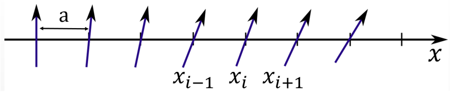

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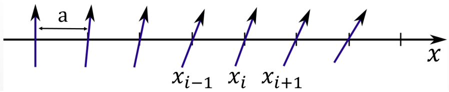

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Spin-systems

for small lattice spacing a we approach for the nearest neighbor

$$\vec{m}_{j+1} \approx \vec{m}_j + a\partial_x \vec{m}_j$$

$$\vec{l}_{j+1} \approx \vec{l}_j + a\partial_x \vec{l}_j$$

$$\sqrt{1 - a^2 \vec{l}_j^2} \approx 1 - \frac{a^2 l_j^2}{2}$$

Interaction term

Heisenberg Hamiltonian:

$$\langle \vec{n}|H|\vec{n}\rangle = Js^2 \sum_j \vec{n}(j)\cdot\vec{n}(j+1) = \frac{Js^2}{2}\sum_j \left([\vec{n}(j)+\vec{n}(j+1)]^2 - 2\right)$$

with our assumptions for small lattice spacing

$$[\vec{n}(j)-\vec{n}(j+1)]^2 - 2 \approx \left[2a\vec{l}_j - (-1)^j a\partial_x \vec{m}_j + a^2\partial_x \vec{l}_j\right]^2, \quad a^2\partial_x\vec{l}_j \to 0$$

$$= [4a^2\vec{l}_j^2 + a^2\vec{m}_j\partial_x^2\vec{m}_j] - 4(-1)^j a^2\vec{l}_j\partial_x\vec{m}_i$$

$$S_{int} = \frac{Js^2}{2}\sum_j \int_0^\beta d\tau \, [4a^2 l_j^2 + a^2(\partial_x m_j)^2] - 4(-1)^j a^2\vec{l}_j\partial_x\vec{m}_j$$

blue marked term vanishes due the alternating sum

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$$\vec{A}(\vec{n_j})\cdot\partial_\tau \vec{n_j} = (-1)^j \vec{A}\partial_\tau \vec{m_j} - a\vec{l_j}(\vec{m_j}\times\partial_\tau \vec{m_j}) \quad + \underbrace{a\partial_\tau(\vec{A}\cdot\vec{l})}_{\to 0}$$

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Partition function

$$Z = \int D\vec{n} \exp[\underbrace{-is \sum_j \int_0^\beta d\tau((-1)^j \vec{A}\partial_\tau \vec{m}_j - a\vec{l}_j(\vec{m}_j \times \partial_\tau \vec{m}_j))}_{\text{Berry phase } S_B}$$

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Continuum limit

suppose $x_\varepsilon = x_0 + \varepsilon \cdot a$

$$\sum_\varepsilon a f(x_\varepsilon) \to \int_{x_0}^{x_1} f(x) dx$$

$$\sum_{j=1}^{N} (-1)^j f(x_j) = \sum_{j=1}^{N/2} f(x_{2j}) - \sum_{j=1}^{N/2} f(x_{2j-1})$$

$$= \frac{1}{2} \sum_{j=1}^{N/2} 2a \underbrace{\frac{f(x_{2j}) - f(x_{2j-1})}{a}}_{\partial_x f(x)} = \frac{1}{2} \int dx \partial_x f(x)$$

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Continuum limit

- sum $\rightarrow$ integral
- using periodic boundary conditions
- apply that $\vec{\nabla} \times \vec{A} = \vec{n}$

$$S[\vec{n}] \approx \int dx \int_0^\beta d\tau$$

$$\left[-i\frac{s}{2}\vec{m}(\partial_x \vec{m} \times \partial_\tau \vec{m}) + \frac{Js^2}{2}(\partial_x \vec{m})^2 - s\vec{l}(\vec{m} \times \partial_\tau \vec{m}) + 2Js^2\vec{l}^2 \right]$$

Gaussian integration

we are only interested in long range order
$\rightarrow$ integrate over the fast fluctuation $\vec{l}$ in the partition function

$$Z = \int D\vec{l} D\vec{m} e^{-iS[\vec{l},\vec{m}]}$$

saddle point method:

$$\int_{-\infty}^{\infty} dl \; e^{if(l)} \approx e^{if(l_0)} \int_{-\infty}^{\infty} dl \; \exp \frac{i}{2} f''(l_0)(l - l_0)^2$$
$$\approx e^{if(l_0)} \sqrt{\frac{2\pi i}{f''(l_0)}}$$

calculate l_0 from $f'(l) = 0 \rightarrow f(l)$ changes slowly around this point

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only red marked part depends on $\vec{l}$

$$-\underbrace{2Js^2}_{v_s}\vec{l}^2 + \vec{l}\underbrace{(\vec{m}\times\partial_x\vec{m})}_{y}s$$

$$\rightarrow f(l) = -v_s l^2 + lys$$

$$\text{for } f'(l) = 0 \rightarrow l_0 = s\frac{y}{2v_s}$$

Result

Partition-function:

$$Z = \sqrt{\frac{4\pi}{v_s}} \int D\vec{m} e^{-\int d\tau dx \mathcal{L}(x,\tau)}$$

$$\mathcal{L} = \underbrace{\frac{1}{2g} \cdot [\frac{1}{v_s}(\partial_\tau \vec{m})^2 + v_s \cdot (\partial_x \vec{m})^2]}_{\mathcal{L}_\sigma} - \underbrace{i\frac{s}{4} \cdot \varepsilon^{\mu\nu} \vec{m}(\partial_\mu \vec{m} \times \partial_\nu \vec{m})}_{\mathcal{L}_T}$$

where $\mathcal{L}_\sigma$ is the same result, which we get from the non-linear-sigma model,
$\mathcal{L}_T$ is the topological term

Topological term

$$
\begin{aligned}
& i\frac{s}{4}\int d\tau dx \varepsilon^{\mu\nu}\vec{m}(\partial_\mu \vec{m} \times \partial_\nu \vec{m}) \\
&= i2\pi s \frac{1}{\pi}\int d\tau dx \varepsilon^{\mu\nu}\vec{m}(\partial_\mu \vec{m} \times \partial_\nu \vec{m}) \\
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- there is a topological term which depends on the value of s

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- there is a topological term which depends on the value of s
- $\mathcal{Q}$ is the winding number which was discussed in the talk of Andreas Löhle, $\mathcal{Q}\in\mathbb{Z}$
  - for integer spin: $e^{-i2\pi s\mathcal{Q}}=1$
  - for half-integer spin: $e^{-i2\pi s\mathcal{Q}}=(-1)^{\mathcal{Q}}$
- the topological term has an effect only for half-integer-spins in one D

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Results

- The one-dimension Heisenberg antiferromagnet can be described by using the path-integral-formalism and leads to same result as the NLσM plus topological term
- we proved Haldane's conjecture:
  - NLσM with topological term for half-integer spin $\rightarrow$ linear dispersion relation was proved by Bethe Ansatz for $s = \frac{1}{2}$
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