Field-theory for the quantum Heisenberg antiferromagnet in one dimension

Seminar: Quantum field-theory on low dimensional systems

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# Table of contents

1. **Introduction**
   - Motivation
   - Heisenberg model

2. **Recapitulation**
   - Path-integral-formalism

3. **Effective action of antiferromagnets**
   - Path-integral

4. **Conclusion**
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## Motivation

- why?
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- e.g. the dispersion relation of spin-wave-excitation of antiferromagnets
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• e.g. the dispersion relation of spin-wave-excitation of antiferromagnets
the dispersion relation of long-wavelength spin-wave excitations for antiferromagnetic systems:

- different approaches calculate the dispersion-relation e.g. by 2nd quantisation or Bethe-ansatz
- the result is a linear dispersion relation for \( s = \frac{1}{2} \)
- it is also a linear dispersion relation for the high spin limit \( s \gg 1 \)
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- \( \rightarrow \) why should there be an other behavior for \( s=1 \)?
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Motivation: experimental measurement

**fig.**: Neutron scattering for $S = \frac{1}{2}$ and $S = 1$


- Measurement of the dispersion relation by neutron scattering
- dashed line: $s = \frac{1}{2} \rightarrow$ massless Dirac-particle
- pointed line: $s = 1 \rightarrow$ spontaneous mass generation
Motivation

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- we get this topological term out of the path-integral formalism
Heisenberg model

- named after Werner Heisenberg
- one approach to describe (anti-)ferromagnetic systems
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- Spin is a quantum mechanical observable $\vec{S}$

$$H = -J \sum_{i=1}^{N} \vec{S}_i \vec{S}_{i+1}$$
Heisenberg model

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- one approach to describe (anti-)ferromagnetic systems
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\[
H = -J \sum_{i=1}^{N} \vec{S}_i \vec{S}_{i+1}
\]
Path-integral-formalism

From the talk of Jan Lotze:

- **Partition-function:**

\[
Z = \int D\vec{n} \delta(n^2 - 1) \exp(-S[\vec{n}]),
\]

\[
= \int D\vec{n} \delta(n^2 - 1) \exp \left[ -\int_0^\beta d\tau \left( \langle \vec{n}| \frac{\partial}{\partial \tau} |\vec{n}\rangle + \langle \vec{n}|H|\vec{n}\rangle \right) \right]
\]

- **Coherent states:**

\[|\vec{n}\rangle = e^{-i\theta \vec{m} \cdot \vec{S}} |s, -s\rangle,\]

\[\langle \vec{n}| \hat{\vec{S}} |\vec{n}\rangle = -s \vec{n}, \quad \vec{n}^2 = 1\]
Path-integral-formalism

- kinetic term

\[ \int_0^\beta d\tau \langle \vec{n} \rvert \frac{\partial}{\partial \tau} \rvert \vec{n} \rangle = -i s \int_0^\beta d\tau [1 - \cos \theta(\tau)] \varphi(\tau) \]

where we assume \( \int_{\partial \Sigma} \vec{A} \, d\vec{n} = \int_{\Sigma} (\nabla \times \vec{A}) \cdot \vec{n} \, df = \Omega \),

where \( \vec{A} \) is a vector potential on the unit sphere with \( \nabla \times \vec{A} = \vec{n} \)

- use periodic boundary conditions
- \( \partial \Sigma \) is a line integral enclosing the surface on the sphere
Path-integral-formalism

\[ Z = \int D\vec{n} \exp \left( -i s \sum_j \int_0^\beta d\tau (\vec{A}(\vec{n}_j) \cdot \partial_\tau \vec{n}_j) - \int_0^\beta d\tau \langle \vec{n} | H | \vec{n} \rangle \right) \]

- Heisenberg Hamiltonian:

\[ \langle \vec{n} | H | \vec{n} \rangle = J \sum_j \langle \vec{n} | \vec{S}_j \cdot \vec{S}_{j+1} | \vec{n} \rangle = Js^2 \sum_j \vec{n}(j) \cdot \vec{n}(j + 1) \]

for nearest neighbor interaction
Path-integral-formalism

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for nearest neighbor interaction
Spin-systems

\[ \vec{n}_j = (-1)^j \vec{n}_{j+1} \text{ and with } \, a \text{ as lattice spacing} \]

- but the spins are staggered
- divide \( \vec{n} \) into a slowly varying part \( \vec{m} \) and a small but fast fluctuation part \( \vec{l} \)

\[ \vec{n}_j = (-1)^j \sqrt{1 - a^2 l_j^2} \, \vec{m}_j + a \vec{l}_j \]
Spin-systems

- $\vec{n}_j = (-1)^j \vec{n}_{j+1}$ and with "a" as lattice spacing
- but the spins are staggered
  divide $\vec{n}$ into a slowly varying part $\vec{m}$ and a small but fast fluctuation part $\vec{l}$

\[
\vec{n}_j = (-1)^j \sqrt{1 - a^2 l_j^2} \vec{m}_j + a \vec{l}_j
\]

still apply

\[
\vec{n}_j^2 = \vec{m}_j^2 = 1, \quad \vec{m}_j \cdot \vec{l}_j = 0
\]
Spin-systems

- \( \vec{n}_j = (-1)^j \vec{n}_{j+1} \) and with \( \text{``a''} \) as lattice spacing
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still apply

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Spin-systems

for small lattice spacing $a$ we approach for the nearest neighbor

$$\vec{m}_{j+1} \approx \vec{m}_j + a \partial_x \vec{m}_j$$

$$\vec{l}_{j+1} \approx \vec{l}_j + a \partial_x \vec{l}_j$$

$$\sqrt{1 - a^2 \vec{l}_j^2} \approx 1 - \frac{a^2 \vec{l}_j^2}{2}$$
Interaction term

Heisenberg Hamiltonian:

\[ \langle \vec{n} | H | \vec{n} \rangle = J s^2 \sum_j \vec{n}(j) \cdot \vec{n}(j + 1) = \frac{J s^2}{2} \sum_j ([\vec{n}(j) + \vec{n}(j + 1)]^2 - 2) \]

with our assumptions for small lattice spacing

\[ ([\vec{n}(j) - \vec{n}(j + 1)]^2 - 2 \approx \left[ 2a \vec{l}_j - (\text{\color{blue}1})^j a \partial_x \vec{m}_j + a^2 \partial_x^2 \vec{l}_j \right]^2, \quad a^2 \partial_x \vec{l}_j \rightarrow 0 \]

\[ = [4a^2 \vec{l}_j^2 + a^2 \vec{m}_j \partial_x^2 \vec{m}_j] - 4(\text{\color{blue}1})^j a^2 \vec{l}_j \partial_x \vec{m}_i \]

\[ S_{\text{int}} = \frac{J s^2}{2} \sum_j \int_0^\beta d\tau \left[ 4a^2 \vec{l}_j^2 + a^2 (\partial_x m_j)^2 \right] - 4(\text{\color{blue}1})^j a^2 \vec{l}_j \partial_x \vec{m}_j \]

\text{blue marked term vanishes due the alternating sum.}
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blue marked term vanishes due the alternating sum
Kinetic term

\[ Z = \int D\vec{n} \exp[-is \sum_j \int_0^\beta d\tau (\vec{A}(\vec{n}_j) \cdot \partial_\tau \vec{n}_j)] \]

Berry phase \( S_B \)

\[ -\frac{Js^2}{2} \sum_j \int_0^\beta d\tau \left[ 4a^2 l_j^2 + a^2 (\partial_x m_j)^2 \right] \]

Interaction term \( S_{int} \)

Berry Phase:

\[ S_B = is \sum_j \int_0^\beta d\tau (\vec{A}(\vec{n}_j) \cdot \partial_\tau \vec{n}_j) \]

- expand \( \vec{A}(\vec{n}_j) \) in \( a \)
- expand \( \partial_\tau \vec{n}_j \) in \( a \)
**Kinetic term**

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Z = \int D\vec{n} \exp\left[ -is \sum_j \int_0^\beta d\tau (\vec{A}(\vec{n}_j) \cdot \partial_\tau \vec{n}_j) \right]
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- **Berry phase** $S_B$

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S_B = is \sum_j \int_0^\beta d\tau (\vec{A}(\vec{n}_j) \cdot \partial_\tau \vec{n}_j)
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- **Interaction term** $S_{int}$

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- \frac{J s^2}{2} \sum_j \int_0^\beta d\tau \left[ 4a^2 l_j^2 + a^2 (\partial_x m_j)^2 \right]
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**Berry Phase:**

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- expand $\vec{A}(\vec{n}_j)$ in $a$
- expand $\partial_\tau \vec{n}_j$ in $a$
Kinetic term

- using the assumption for staggered spins
- Taylor-expansion in $a$

\[
A_\mu[\vec{n}] = A_\mu[\vec{m}, \vec{l}] = A_\mu[(-1)^j \vec{n}]
\]
\[
= A_\mu \left[ \sqrt{1 - a^2 l^2} \cdot \vec{m} + (-1)^j a \vec{l} \right]
\]
\[
\approx A_\mu(\vec{m}) + \partial_\nu A_\mu(\vec{m})[(-1)^j a l^{\nu}] + O(a^2)
\]

\[
\partial_\tau n^\mu = (-1)^j \partial_\tau m^\mu + a \partial_\tau l^\mu + O(a^2)
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we apply again: $\vec{\nabla} \times \vec{A} = \vec{n}$ and use $\int_a^b d\tau \frac{\partial f}{\partial \tau} = f(b) - f(a)$

\[
\vec{A}(\vec{n}_j) \cdot \partial_\tau \vec{n}_j = (-1)^j \vec{A} \partial_\tau \vec{m}_j - a \vec{l}_j(\vec{m}_j \times \partial_\tau \vec{m}_j) + a \partial_\tau (\vec{A} \cdot \vec{l})
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\[
\vec{A}(\vec{n}_j) \cdot \partial_\tau \vec{n}_j = (-1)^j \vec{A} \partial_\tau \vec{m}_j - a l_j (\vec{m}_j \times \partial_\tau \vec{m}_j) + a \partial_\tau (\vec{A} \cdot \vec{l}) \rightarrow 0
\]
Partition function

\[ Z = \int D\bar{n} \exp[-is \sum_j \int_0^\beta d\tau ((-1)^j \mathbf{\dot{A}} \partial_\tau \mathbf{m}_j - a\mathbf{l}_j (\mathbf{m}_j \times \partial_\tau \mathbf{m}_j))} \]

\[ - \frac{Js^2}{2} \sum_j \int_0^\beta d\tau \left[ 4a^2 l_j^2 + a^2 (\partial_x m_j)^2 \right] \]

Berry phase \( S_B \)

interaction term \( S_{int} \)
Continuum limit

suppose $x_\varepsilon = x_0 + \varepsilon \cdot a$

$$\sum_{\varepsilon} a f(x_\varepsilon) \to \int_{x_0}^{x_1} f(x) dx$$

$$\sum_{j=1}^{N} (-1)^j f(x_j) = \sum_{j=1}^{N/2} f(x_{2j}) - \sum_{j=1}^{N/2} f(x_{2j-1})$$

$$= \frac{1}{2} \sum_{j=1}^{N/2} 2a \frac{f(x_{2j}) - f(x_{2j-1})}{a} = \frac{1}{2} \int dx \partial_x f(x)$$
Continuum limit

suppose \( x_\varepsilon = x_0 + \varepsilon \cdot a \)

\[
\sum_{\varepsilon} \alpha f(x_\varepsilon) \rightarrow \int_{x_0}^{x_1} f(x) \, dx
\]

\[
\sum_{j=1}^{N} (-1)^j f(x_j) = \sum_{j=1}^{N/2} f(x_{2j}) - \sum_{j=1}^{N/2} f(x_{2j-1})
\]

\[
= \frac{1}{2} \sum_{j=1}^{N/2} 2a \left( \frac{f(x_{2j}) - f(x_{2j-1})}{a} \right)
= \frac{1}{2} \int dx \partial_x f(x)
\]
Continuum limit

- sum $\to$ integral
- using periodic boundary conditions
- apply that $\tilde{\nabla} \times \tilde{A} = \tilde{n}$

$$S[\tilde{n}] \approx \int d\mathbf{x} \int_0^\beta d\tau$$
$$\left[ -i \frac{s}{2} \tilde{m}(\partial_x \tilde{m} \times \partial_\tau \tilde{m}) + \frac{Js^2}{2} (\partial_x \tilde{m})^2 - s\tilde{l}(\tilde{m} \times \partial_\tau \tilde{m}) + 2Js^2 \tilde{l}^2 \right]$$
we are only interested in long range order
→ integrate over the fast fluctuation $\vec{I}$ in the partition function

$$Z = \int D\vec{I} D\vec{m} e^{-iS[\vec{I},\vec{m}]}$$

saddle point method:

$$\int_{-\infty}^{\infty} dl \ e^{if(l)} \approx e^{if(l_0)} \int_{-\infty}^{\infty} dl \ \exp \frac{i}{2} f''(l_0)(l - l_0)^2$$

$$\approx e^{if(l_0)} \sqrt{\frac{2\pi i}{f''(l_0)}}$$

calculate $l_0$ from $f'(l) = 0 \rightarrow f(l)$ changes slowly around this point
Gaussian integration

\[ S[\mathbf{n}] \approx \int d\mathbf{x} \int_{0}^{\beta} d\tau \]

\[ \left[ -i\frac{s}{2} \mathbf{m}(\partial_{x} \mathbf{m} \times \partial_{\tau} \mathbf{m}) + \frac{Js^{2}}{2} (\partial_{x} \mathbf{m})^{2} - (s\mathbf{l}(\mathbf{m} \times \partial_{x} \mathbf{m}) - 2Js^{2}\mathbf{l}^{2}) \right] \]

only red marked part depends on \( \mathbf{l} \)

\[ -2Js^{2}\mathbf{l}^{2} + \mathbf{l}(\mathbf{m} \times \partial_{x} \mathbf{m}) s \]

\[ \nu_{s} \]

\[ y \]

\[ \rightarrow f(l) = -\nu_{s}l^{2} + lys \]

for \( f'(l) = 0 \rightarrow l_{0} = s \frac{y}{2\nu_{s}} \]
Result

Partition-function:

\[
Z = \sqrt{\frac{4\pi}{v_s}} \int D\vec{m} e^{-\int d\tau d\mathbf{x} \mathcal{L}(\mathbf{x},\tau)}
\]

\[
\mathcal{L} = \frac{1}{2g} \cdot \left[ \frac{1}{v_s} (\partial_\tau \vec{m})^2 + v_s \cdot (\partial_\mathbf{x} \vec{m})^2 \right] - i s \frac{\varepsilon^{\mu
\nu}}{4} \vec{m} (\partial_\mu \vec{m} \times \partial_\nu \vec{m})
\]

where \(\mathcal{L}_\sigma\) is the same result, which we get from the non-linear-sigma model,
\(\mathcal{L}_T\) is the topological term
### Topological term

\[
i \frac{s}{4} \int d\tau dx \varepsilon^{\mu \nu} \mathbf{m} (\partial_\mu \mathbf{m} \times \partial_\nu \mathbf{m})
\]

\[
= i 2\pi s \frac{1}{\pi} \int d\tau dx \varepsilon^{\mu \nu} \mathbf{m} (\partial_\mu \mathbf{m} \times \partial_\nu \mathbf{m})
\]

\[
= i 2\pi s Q
\]

- there is a topological term which depends on the value of \(s\)
Topological term

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\[ = i2\pi s Q \]

- there is a topological term which depends on the value of \( s \)
- \( Q \) is the winding number which was discussed in the talk of Andreas Löhle, \( Q \in \mathbb{Z} \)
  - for integer spin: \( e^{-i2\pi s Q} = 1 \)
  - for half-integer spin: \( e^{-i2\pi s Q} = (-1)^Q \)
- the topological term has an effect only for half-integer-spins in one D
there is a topological term which depends on the value of $s$

$Q$ is the winding number which was discussed in the talk of Andreas Löhle, $Q \in \mathbb{Z}$

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the topological term has an effect only for half-integer-spins in one D
The one-dimension Heisenberg antiferromagnet can be described by using the path-integral-formalism and leads to same result as the NL$\sigma$M plus topological term

we proved Haldane’s conjecture:

- $NL\sigma$M with topological term for half-integer spin $\rightarrow$ linear dispersion relation was proved by Bethe Ansatz for $s = \frac{1}{2}$
- $NL\sigma$M without topological term for integer spin $\rightarrow$ gap in the dispersion relation
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we can explain the energy-gap of the neutron-scattering for $s=1$
The one-dimension Heisenberg antiferromagnet can be described by using the path-integral-formalism and leads to the same result as the NL\(\sigma\)M plus topological term. We proved Haldane’s conjecture:

- NL\(\sigma\)M with topological term for half-integer spin \(s = \frac{1}{2}\) → linear dispersion relation was proved by Bethe Ansatz.
- NL\(\sigma\)M without topological term for integer spin \(s = \frac{1}{2}\) → gap in the dispersion relation.

We can explain the energy-gap of the neutron-scattering for \(s = 1\).

Antiferromagnetism in two dimension by Wolfgang Voesch.
The one-dimensional Heisenberg antiferromagnet can be described by using the path-integral-formalism and leads to the same result as the NL$\sigma$M plus topological term. We proved Haldane’s conjecture:

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