Free Space QED and Emergent Universal Dynamics with a Single Rydberg Superatom

University of Stuttgart
Germany

Jan Kumlin[1], Sebastian Hofferberth[2], and Hans Peter Büchler[1]

[1]Institute for Theoretical Physics III, University of Stuttgart, Germany

[2]Department of Physics, Chemistry and Pharmacy, University of Southern Denmark, Odense, Denmark

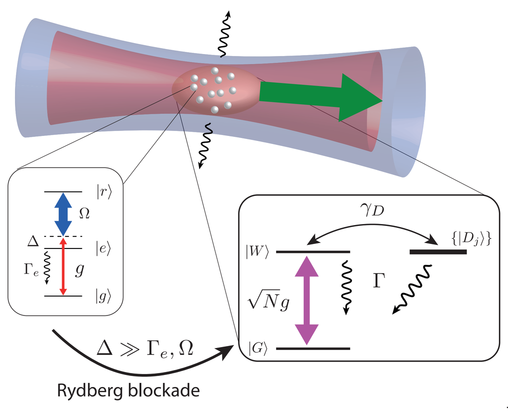
SDU

Introduction

The interaction of a single photon with an individual two-level system is the textbook example of quantum electrodynamics. Achieving strong coupling in this system has so far required confinement of the light field inside resonators of waveguides. Here, we present the theoretical background of the experimental demonstration of strong coherent coupling between a single Rydberg superatom and a propagating light pulse containing only a few photons [1].
We also study the influence of the coherent exchange interactions between the atoms inside the superatom mediated by exchange of virtual photons in a simple one-dimensional setup. In contrast to the naive expectation of fast dephasing of the bright state due to exchange interactions, we find oscillations and dephasing on a time scale that grows with the number of atoms inside the superatom. Further, we analyze this behaviour analytically using perturbation theory finding excellent agreement with our numerical simulations.

Free Space QED with a Single Rydberg Superatom [1]

Setup and Model



- N driven Rydberg atoms
- Rydberg blockade leads to collective states

$S^+(\mathbf{r}) = \psi_r^\dagger(\mathbf{r})\psi_g(\mathbf{r})$

bright state: $|W\rangle = \frac{1}{\sqrt{N}}\int d\mathbf{r}\, u(\mathbf{r}) S^+(\mathbf{r})|G\rangle$

dark states: $|D_j\rangle$ with $\langle W|D_j\rangle = 0$

➤ directional emission into forward propagating mode due to collective excitations (averaged over atomic positions)

$$\bar\Gamma_{\mathbf{q},\mu} = \frac{2\pi g^2}{\hbar^2}\delta(\omega - c|\mathbf{q}|)|c^\mu_{\mathbf{q}}|^2\left[1 + \frac{1}{N}\left|\int d\mathbf{r}\, u(\mathbf{r}) n(\mathbf{r})\right|^2\right]$$

normal decay rate into free space

collective enhancement $\sim N$ of decay into specific mode

➤ enhanced decay in forward direction with an opening angle

$$\sin^2\theta \lesssim \frac{1}{\pi^2}\frac{\lambda^2}{w_0^2}$$

w_0: waist of Gaussian beam

➤ backscattering suppressed by $e^{-8\pi^2\sigma_z^2/\lambda^2} \to 0$ for a Gaussian density distribution with width $\sigma_z/\lambda \gg 1$

➤ define effective 1D model with coupling $\kappa = \frac{2\pi(N+1)g^2\omega}{A\hbar c}$

$$H = \int \frac{dk}{2\pi}\hbar c k\, a_k^\dagger a_k + \hbar\sqrt{\kappa}\left(E^\dagger(0)\sigma_{GW} + E(0)\sigma_{GW}^\dagger\right)$$

$$\sigma_{\alpha\beta} = |\alpha\rangle\langle\beta|$$

Master Equation and Dynamical Phase Diagram

- derive master equation from effective Hamiltonian by integrating out photonic degrees of freedom for coherent input field $\alpha(t)$

➤ open system's dynamics of single two-level system

$$\partial_t\rho(t) = -i\alpha(t)\sqrt{\kappa}\left[\sigma_{GW} + \sigma_{GW}^\dagger, \rho(t)\right] + \kappa\left(\sigma_{GW}\rho(t)\sigma_{GW}^\dagger - \frac{1}{2}\left\{\sigma_{GW}^\dagger\sigma_{GW}, \rho(t)\right\}\right)$$

➤ intrinsically damped system due to enhanced decay into forward propagating mode (other decay channels can be put in phenomenologically)

- relate outgoing electric field to the dynamics of the atom

$$E(t) = \alpha(t) - i\sqrt{\kappa}\sigma_{GW}(t)$$

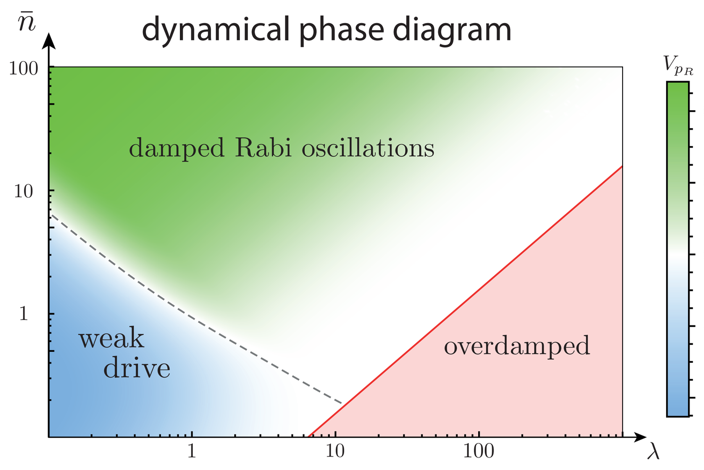
➤ imprint correlations onto outgoing field

- dimensionless parameters of the problem

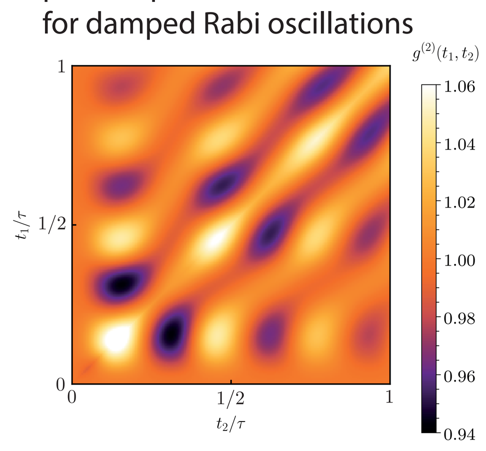
$\lambda = \kappa\tau$ coupling parameter

$\bar n = \alpha^2\tau$ mean photon number

dynamical phase diagram



photon-photon correlations for damped Rabi oscillations



Quantum Regression Theorem [2]

- evaluate unequal-time correlation functions within the Born-Markov approximation
- time evolution of the density matrix

$$\partial_t\rho(t) = \mathcal{L}(t)\rho(t)$$

$$\rho(t) = \mathcal{V}(t,t_0)\rho(t_0) = \mathcal{T}\exp\left(\int_{t_0}^t dt'\,\mathcal{L}(t')\right)\rho(t_0)$$

propagator for density matrix

$$\partial_t\mathcal{V}(t,t_0) = \mathcal{L}(t)\mathcal{V}(t,t_0) \qquad \mathcal{V}(t_0,t_0) = 1$$

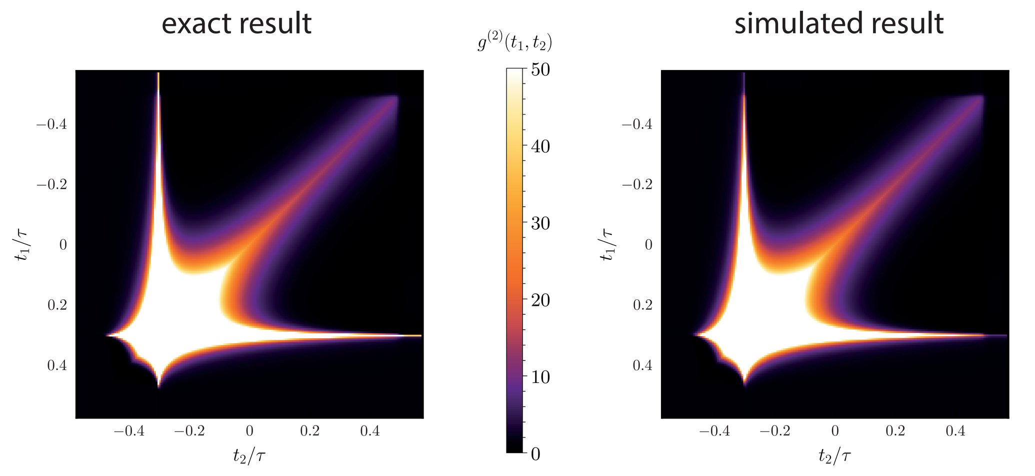
- correlation functions

$$\langle A(t+\tau)B(t)\rangle = \mathrm{Tr}_S\left\{A\mathcal{V}(t+\tau,t)\left[B\rho(t)\right]\right\} \qquad \tau \geq 0$$

$$\langle A(t_1)B(t_2)C(t_2)D(t_1)\rangle = \mathrm{Tr}_S\left\{BC\mathcal{V}(t_2,t_1)\left[D\rho(t_1)A\right]\right\} \quad t_2 \geq t_1$$

- Quantum Regression Theorem is exact for a single two-level emitter [3]
- compare result for 2nd order correlation function with exact solution using Bethe ansatz [4] for $\bar n \ll 1$

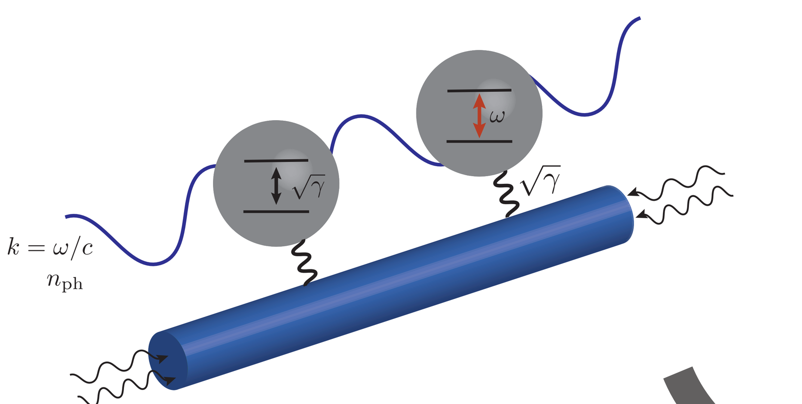
exact result — simulated result



Influence of the Interactions - Emergent Universal Dynamics

Waveguide Model [5]

- We study the influence of interactions between the atoms inside the superatom mediated by exchange of virtual photons within a simple one-dimensional waveguide model. The atomic positions are assumed to be randomly distributed with the density distribution given by $n(x)$ with a characteristic width σ.

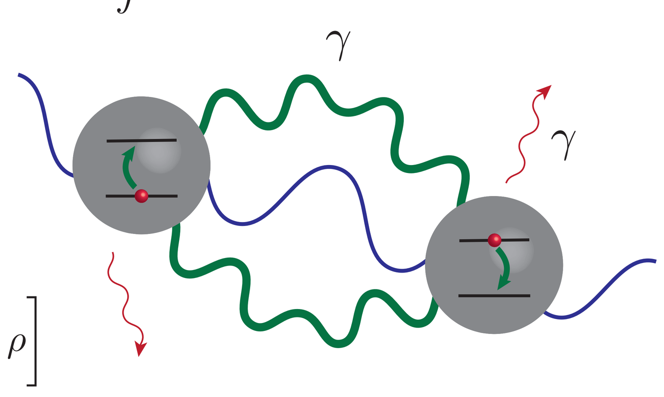


$$H_B = \int \frac{dq}{2\pi}\hbar c q\, a_q^\dagger a_q$$

$$H_{\rm int} = i\hbar\sqrt{\gamma}\int dx\int\frac{dq}{2\pi}a_q^\dagger S^-(x)e^{-iqx} + \text{h.c.}$$

$$H_{\rm sys} = \hbar\sqrt{\gamma n_{\rm ph}}\int dx\, e^{ikx}S^+(x) + \text{h.c.}$$

integrate out photonic degrees of freedom



$$\partial_t\rho = -\frac{i}{\hbar}\left[H_{\rm sys} + \hbar\gamma\int dx\int dy\,\sin(k|x-y|)S^+(x)S^-(y), \rho\right]$$

coherent exchange interaction (Lamb shift)

$$+2\gamma\int dx\int dy\,\cos(k|x-y|)\left(S^-(x)\rho S^+(y) - \frac{1}{2}\left\{S^+(x)S^-(y), \rho\right\}\right)$$

Lindblad dissipator (spontaneous decay)

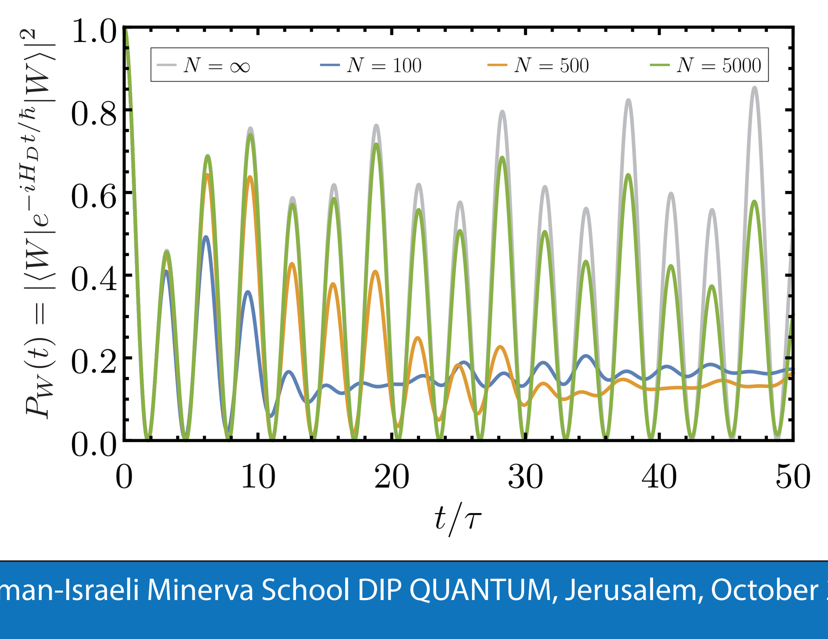
Coherent Dynamics - Exchange Interaction Hamiltonian

➤ expect fast dephasing out of $|W\rangle$ state by coherent exchange Hamiltonian on time scale

$$\tau_D \sim \sqrt{12/N}\gamma$$

➤ numerical simulations of coherent dynamics show oscillations on time scale $\tau = \pi/N\gamma$ which are damped out on time scale

$$\tau_{\rm dp} = \sqrt{N}/\gamma$$



Incoherent Dynamics

➤ enhanced spontaneous emission rate into forward propagating mode

$$\Gamma_F = N\gamma$$

➤ decay into backward propagating mode (after averaging over atomic positions)

$$\bar\Gamma_B = \gamma(N-1)\frac{|n_k|^2}{N^2} + \gamma \approx \gamma$$

FT of density distribution

for smooth density distribution

$$P_W(t) = \begin{cases} [\chi(t/\tau)]^2, & t \ll \tau_{\rm dp} \\ 1/6, & t \gtrsim \tau_{\rm dp}\end{cases}$$

$$\chi(s) = \frac{8}{\pi^2}\sum_{n=0}^\infty\frac{1}{(2n+1)^2}\cos\left(\frac{s}{2n+1}\right)$$

Emergent Universal Dynamics

Analytical Calculation of the Spectrum

- split $H_D = H_+ + H_-$ with $H_\pm = \frac{\hbar\gamma}{2i}\int dx\int dy\,\mathrm{sgn}(x-y)e^{\pm ik(x-y)}S^+(x)S^-(y)$

Spectrum of H_+ and H_-

$$E_\alpha = \frac{\hbar\gamma}{2}\cot\left(\frac{\pi\alpha}{2N}\right) \sim \frac{\hbar\gamma N}{\pi\alpha}$$

$-N \leq \alpha \leq N \quad \alpha$ odd

Eigenstates of $H_\pm$

$$|\alpha,\pm\rangle = \frac{1}{\sqrt{N}}\int dx\, e^{ikx}e^{\mp i\frac{\pi\alpha}{N}\int_{-\infty}^x dz\, n(z)}S^+(x)|G\rangle$$

$$\rightarrow \quad |W\rangle \sim -\sum_\alpha\frac{2}{\pi\alpha}|\alpha,\pm\rangle$$

➤ since only values with small α have significant overlap with $|W\rangle$ and have a characteristic energy scale $E_0 = \hbar\gamma N$, one can perform degenerate perturbation theory for small α

Spectrum in Perturbation Theory

$$E_\pm = E_\alpha\left(\frac{1-2|w_\alpha|^2}{1-|w_\alpha|^2} \pm \frac{\sqrt{\delta_\alpha^2(1-|w_\alpha|^2)+|w_\alpha|^2}}{1-|w_\alpha|^2}\right)$$

$$w_\alpha = \langle\alpha,+|\alpha,-\rangle \neq 0 \text{ in general}$$

$$\delta_\alpha = \langle\alpha,+|H_-|\alpha,+\rangle$$

Two Analytical Limits

$N = \infty$ and $k\sigma \to \infty$

$\overline{w_\alpha} \to 0$

$\overline{\delta_\alpha} \sim c_0/k\sigma + \alpha c_1/(k\sigma)^2 \to 0$

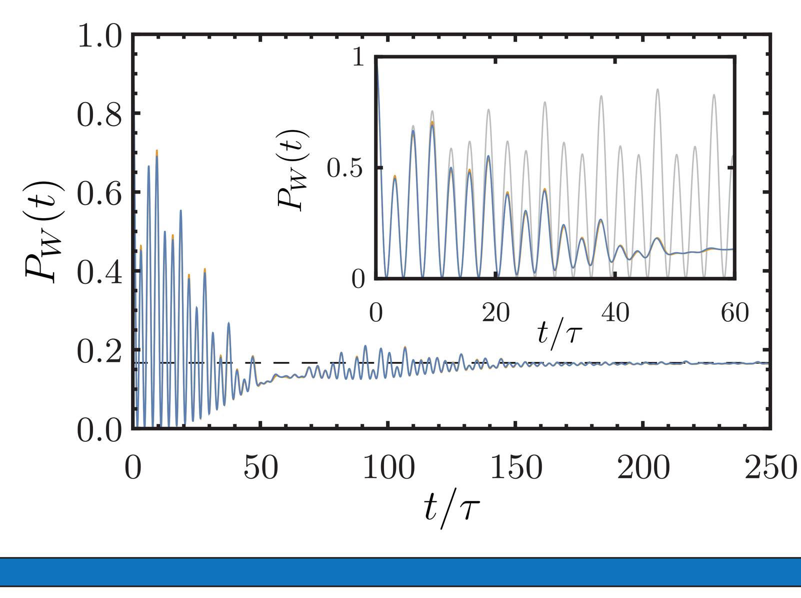
➤ $|\alpha,\pm\rangle$ become eigenstates of full H

N finite and $k\sigma = \infty$

$\Delta E/E_0 \sim 1/\sqrt{N}$

➤ dephasing on time scale $\tau_{\rm dp} = \sqrt{N}\tau$

➤ $|w_\alpha|^2$ is Poisson distributed



➤ analytical calculation of $P_W(t)$

➤ excellent agreement with numerical simulations

➤ saturation at $1/6$ due to equal distribution onto + and - eigenstates

[1] A. Paris-Mandoki et al., Phys. Rev. X **7**, 041010 (2017)
[2] M. Lax, Phys. Rev. **129**, 2342 (1963)
[3] T. Shi et al., Phys. Rev. A **92**, 053834 (2015)
[4] V. I. Yudson and P. Reineker, Phys. Rev. A **78**, 052713 (2008)
[5] H. Pichler et al., Phys. Rev. A **91**, 042116 (2015)

Phys. Rev. X 7, 041010 (2017)

erc European Research Council

kumlin@itp3.uni-stuttgart.de