Strong coupling of a Rydberg superatom to a propagating light mode

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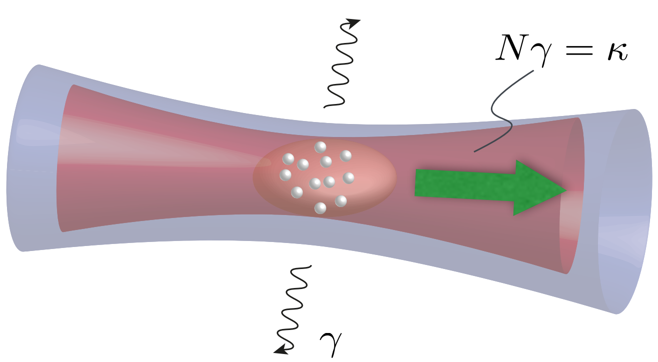
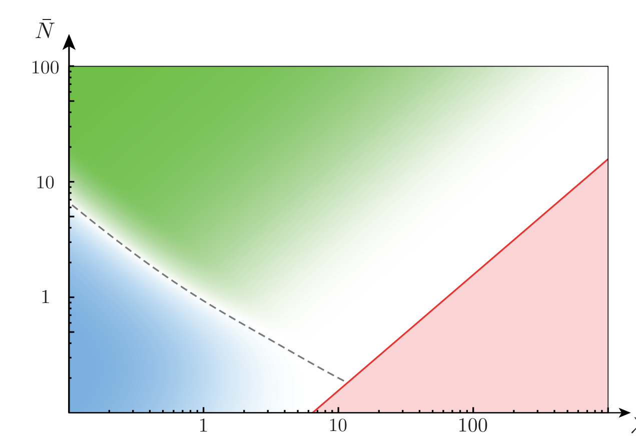


SFB/TRR 21 Workshop
Loveno di Menaggio, Italy
03. - 06.04.2017



CO.CO.MAT
CONTROL OF QUANTUM CORRELATIONS IN TAILORED MATTER
SFB/TRR 21 – STUTTGART, ULM, TÜBINGEN

University of Stuttgart
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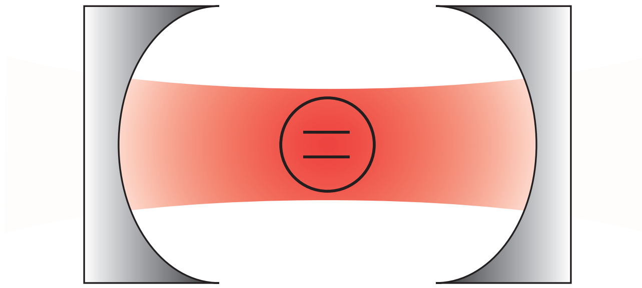
Cavity/Circuit Quantum Electrodynamics

single light-mode + single two-level system (TLS)



Jaynes-Cummings model

*J.M. Raimond et al.,RMP **73**, 565 (2001)*
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✓ *strong coupling between light mode and atom*

✗ *how to get the photon out?*

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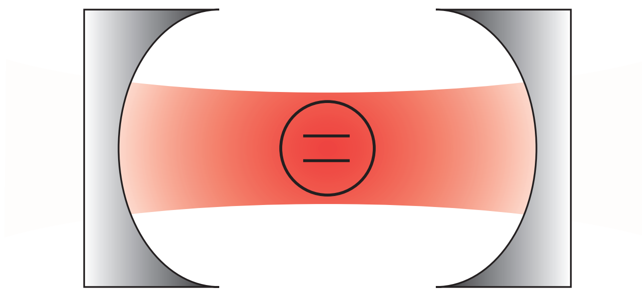

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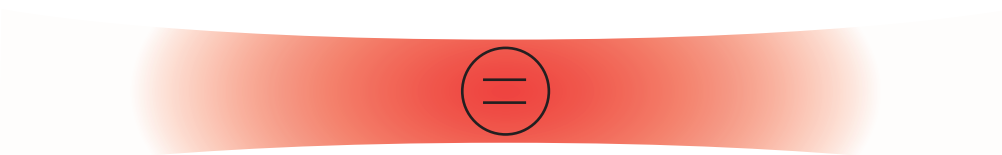
"Free Space" Quantum Electrodynamics



propagating light mode + single TLS

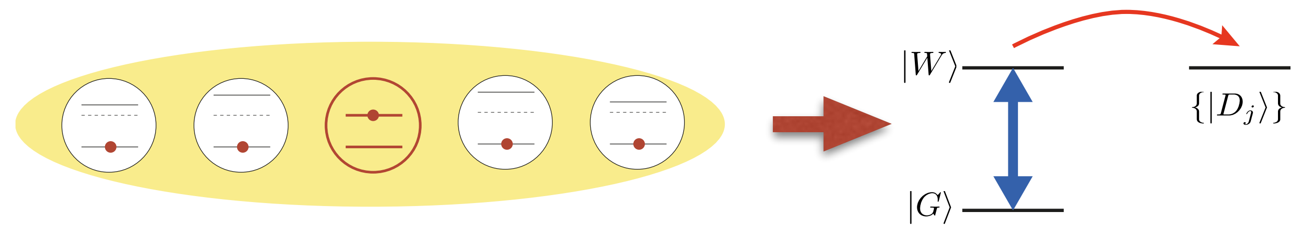
✓ possibility of on-the-fly processing of photonic qubits

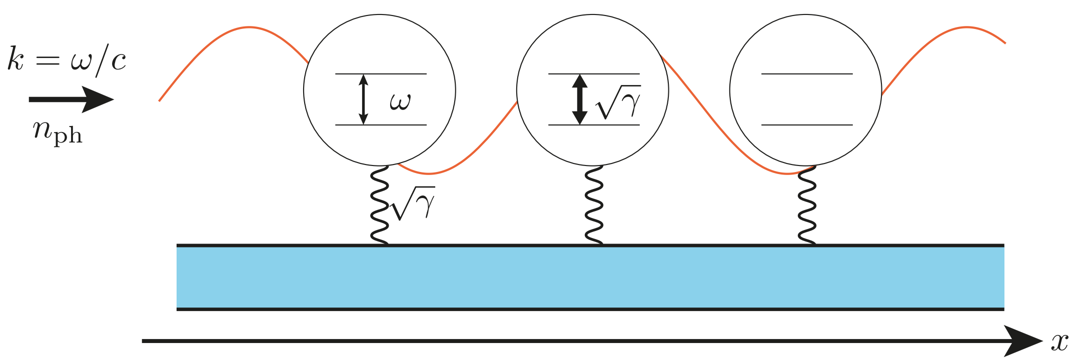
✓ interface between flying and matter qubits

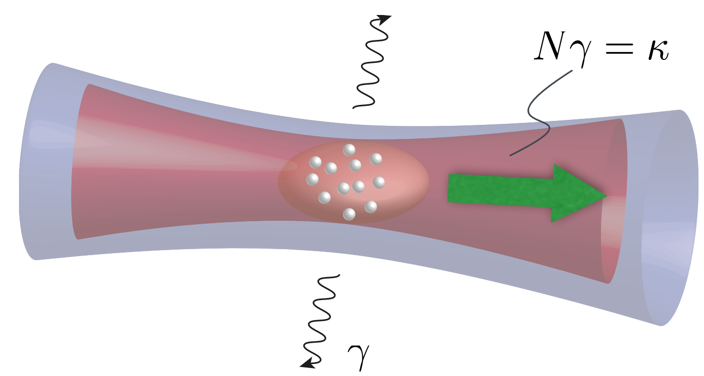
? strong coupling / implementation with Rydberg atoms

Outline

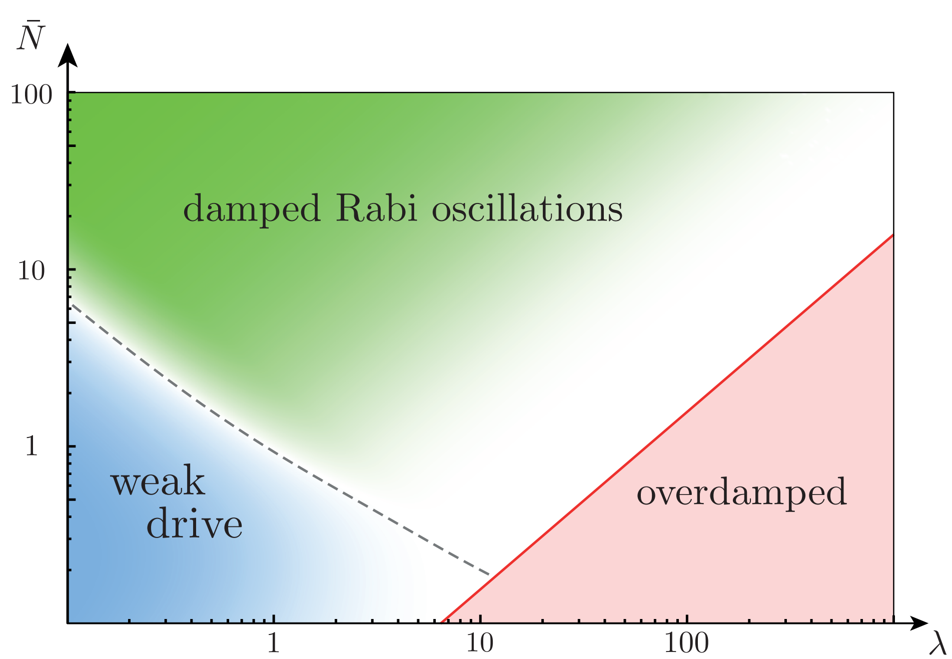
1. Theory and model







$$\partial_t \rho = -\frac{i}{\hbar}[H,\rho] + \gamma \mathcal{D}\rho$$



2. Dynamical phase diagram

Outline

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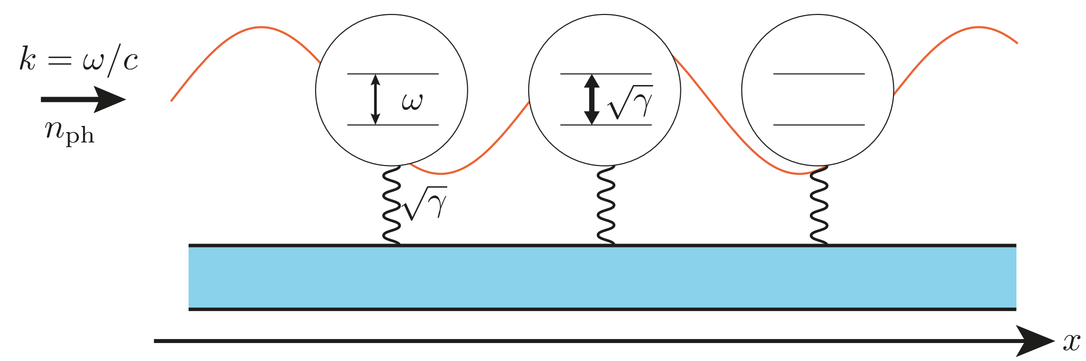

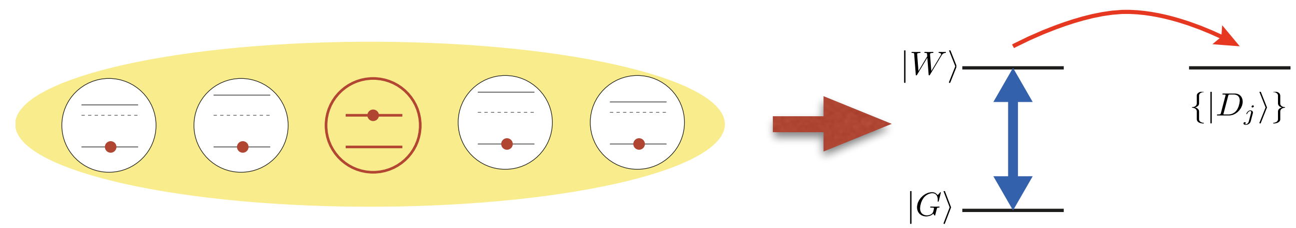

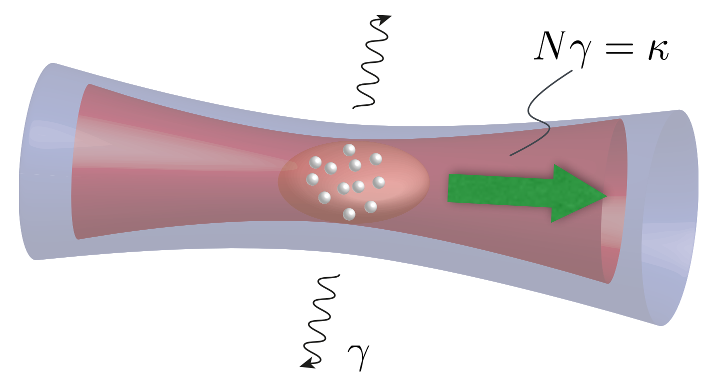

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2. Dynamical phase diagram

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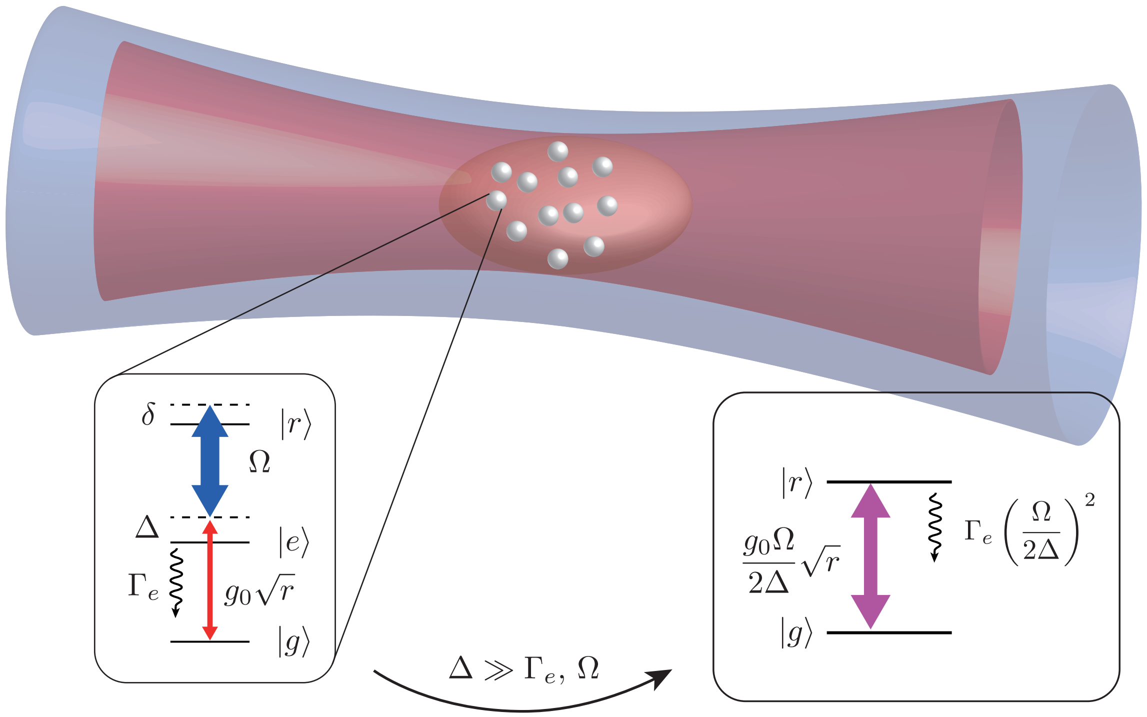
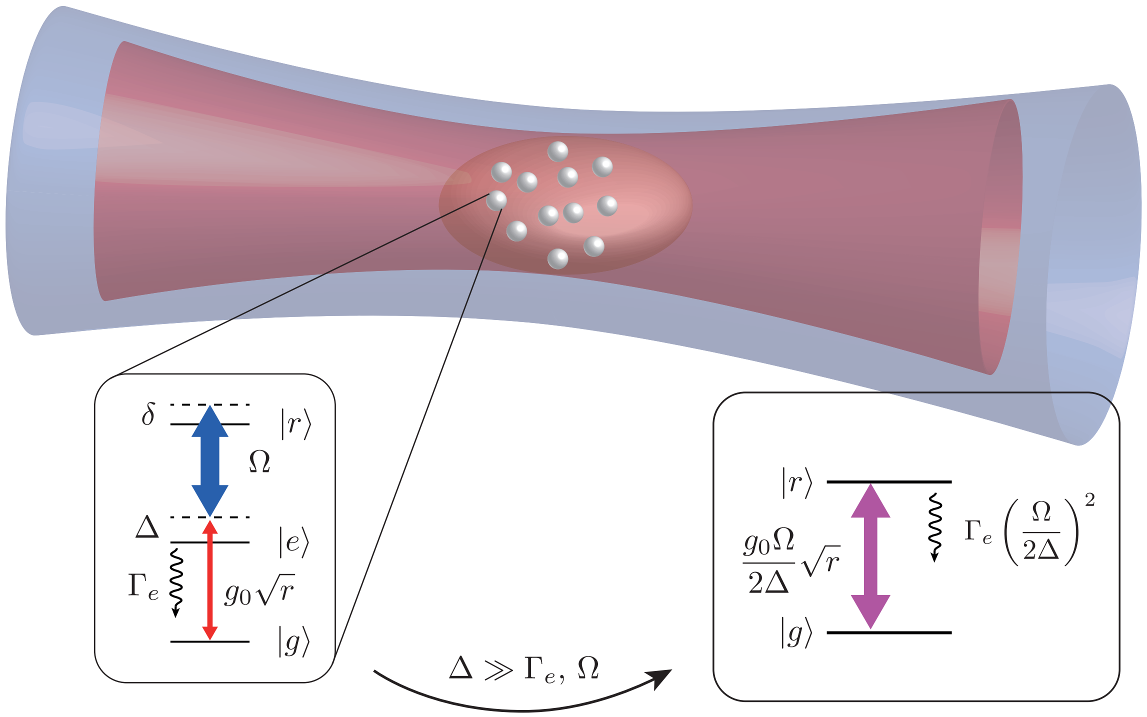


- confined, driven Rydberg atoms
- adiabatic elimination of $|e\rangle$ for $\Delta \gg \Gamma_e,\, \Omega$
  ➡ effective two-level system (TLS)
- single excitation if confinement smaller than blockade radius

Setup



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→ derive master equation for the atomic degrees of freedom

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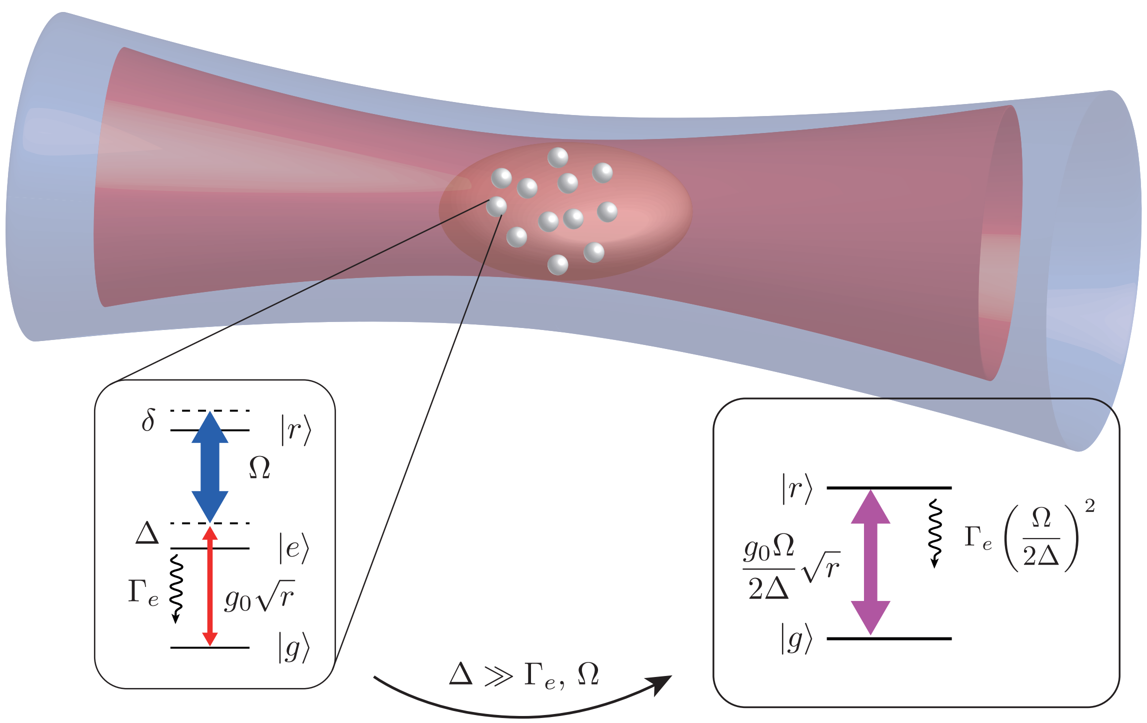

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3D system of TLS

*R. Lehmberg, Phys. Rev. A **2**, 883 (1970)*
*D. Porras/ J. I. Cirac, Phys. Rev. A **78**, 053816 (2008)*

➡ superatom analysis rather complicated and technical

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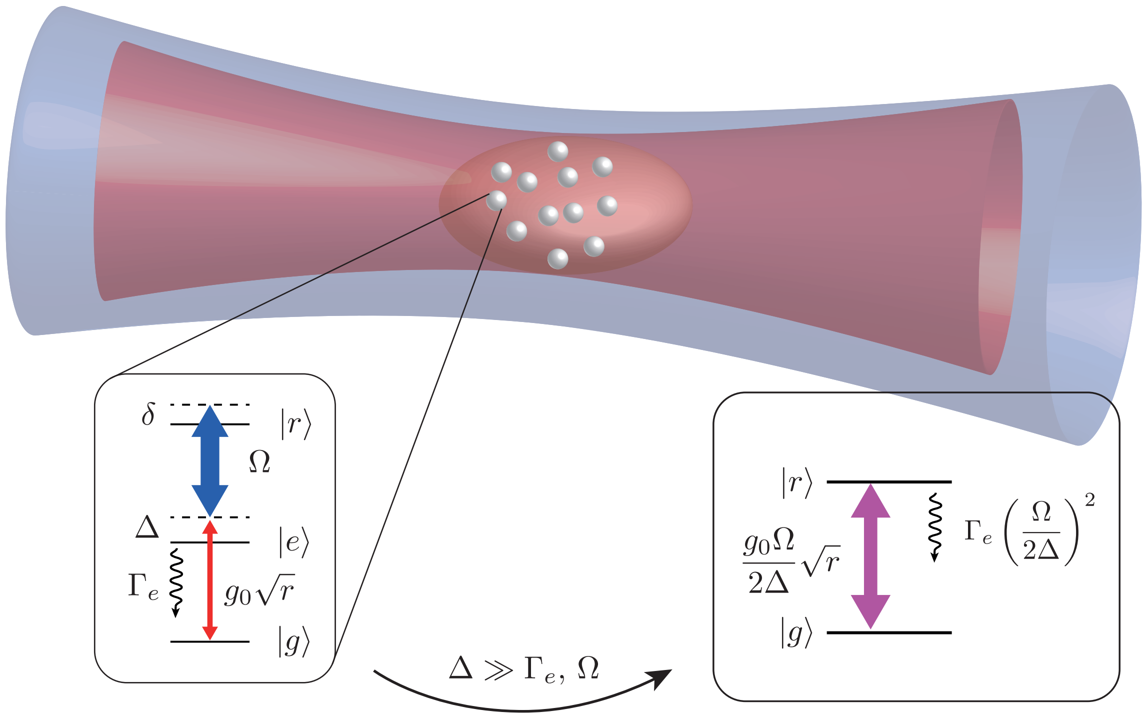

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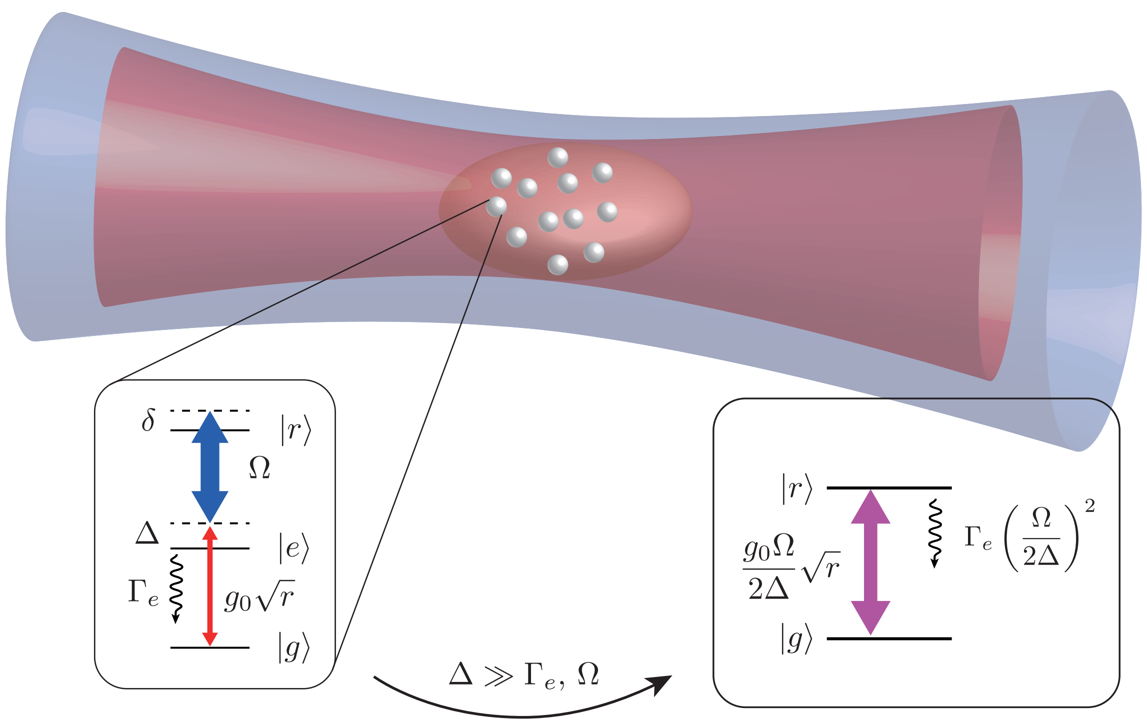

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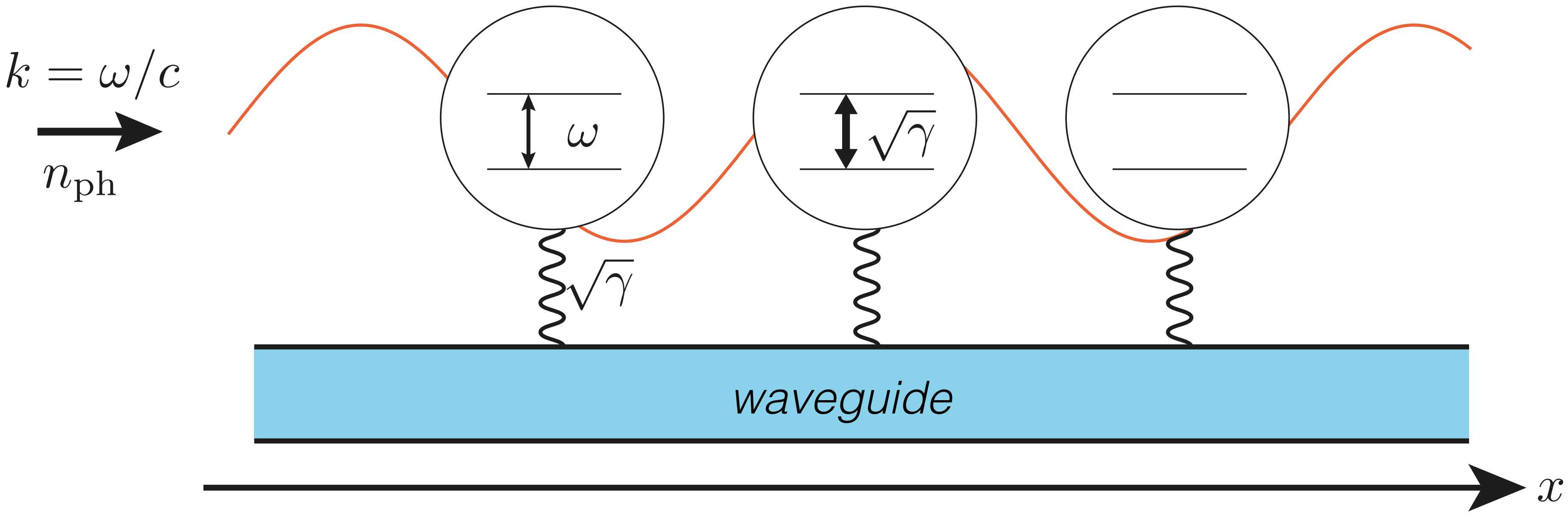
1D system / N TLS coupled to a waveguide *H. Pichler et al., PRA **91**, 042116 (2015)*
*T. Shi et al., PRA **92**, 053834 (2015)*

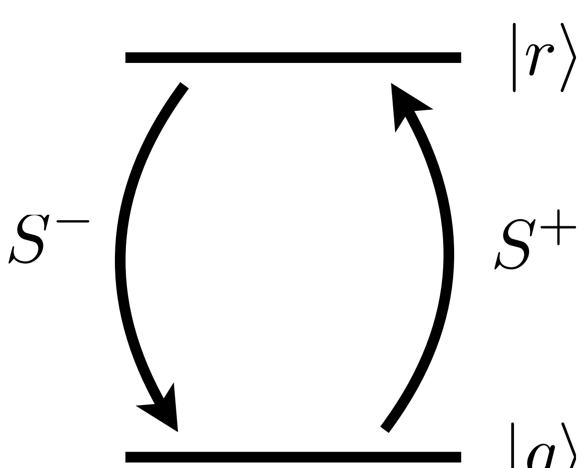
➡ gain understanding in simpler setup

1D waveguide model

H. Pichler et al., PRA **91**, 042116 (2015)
T. Shi et al., PRA **92**, 053834 (2015)



introduce field operators to create Rydberg excitation at position x



$$S^+(x) = \psi_r^\dagger(x)\psi_g(x)$$

$$S^-(x) = \psi_g^\dagger(x)\psi_r(x)$$

$$\left[S^-(x), S^+(y)\right] = \left(\psi_g^\dagger(x)\psi_g(x) - \psi_r^\dagger(x)\psi_r(x)\right)\delta(x-y)$$

1D waveguide model

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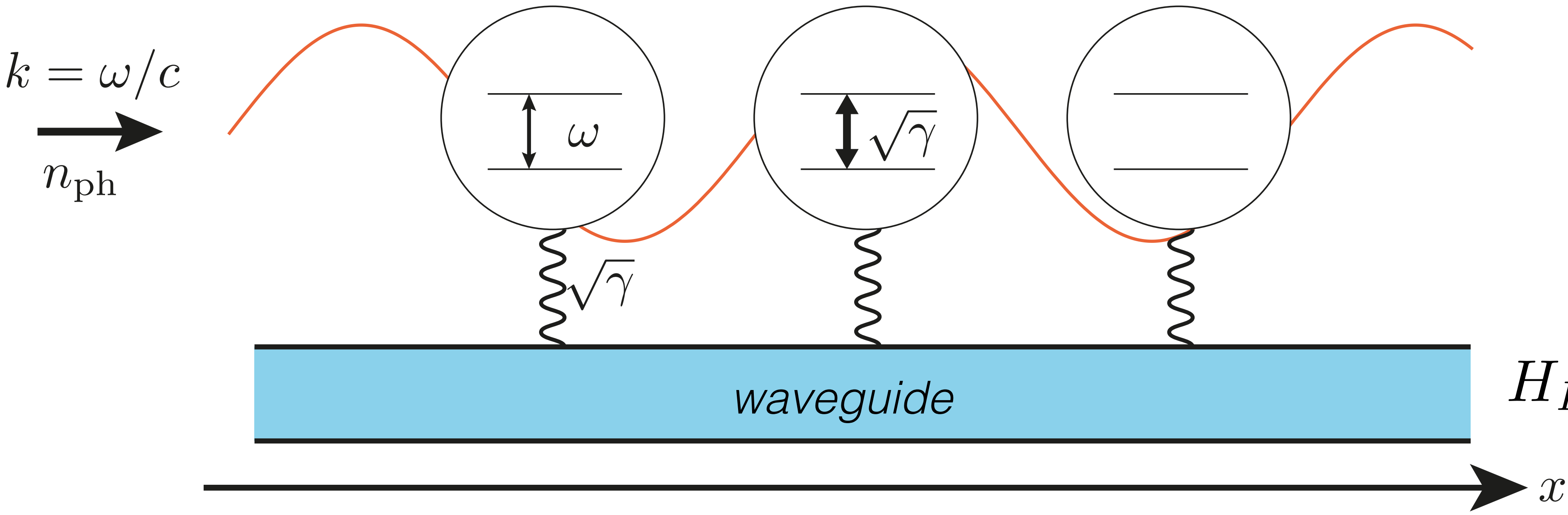

$$H_B = \hbar c \int \frac{dq}{2\pi}\, q\, b_q^\dagger b_q$$

$$H_{\mathrm{sys}} = \hbar\sqrt{\gamma\, n_{\mathrm{ph}}} \int dx\, e^{ikx} S^+(x) \; + \; \mathrm{h.c.}$$

$$H_{\mathrm{int}} = i\hbar\sqrt{\gamma} \int dx \int \frac{dq}{2\pi}\, b_q^\dagger S^-(x) e^{-iqx} \; + \; \mathrm{h.c.}$$

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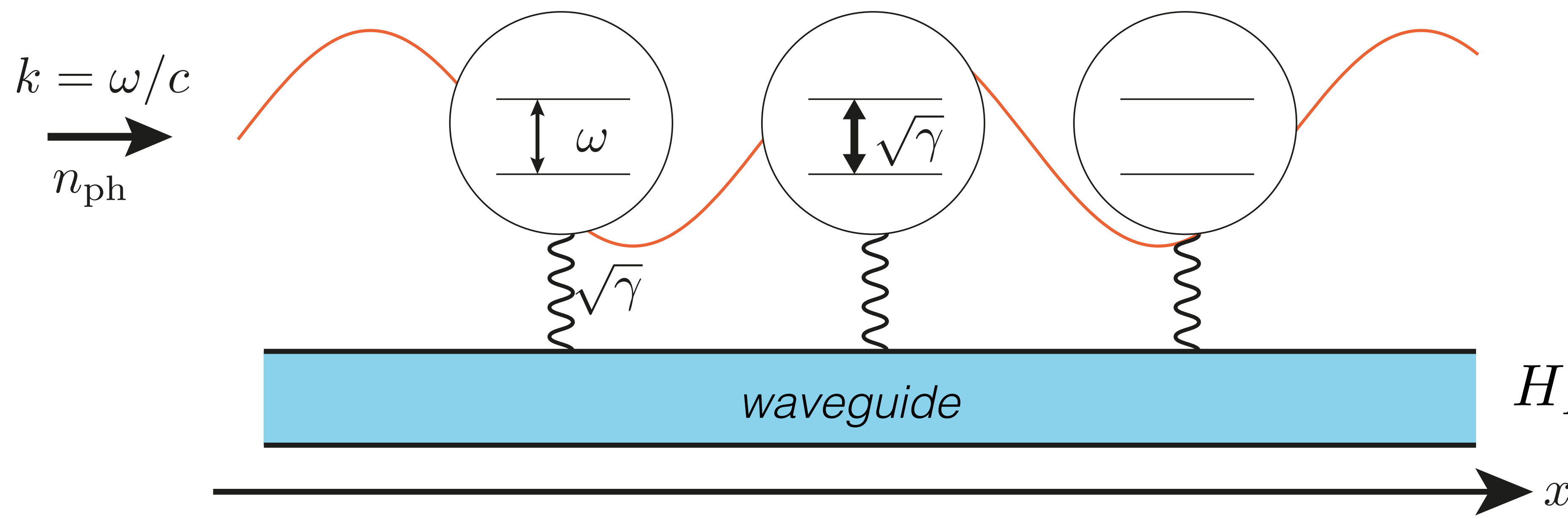

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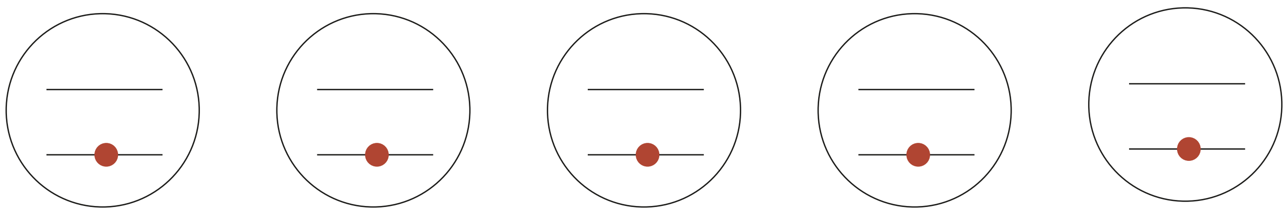
➡ derive master equation for system by integrating out waveguide modes

$$\partial_t \rho = -\frac{i}{\hbar}\left[H_{\mathrm{sys}} + \hbar\gamma \int dx \int dy \sin(k|x-y|) S^+(x) S^-(y), \rho\right]$$
$$+ 2\gamma \int dx \int dy \cos(k|x-y|) \left(S^-(x)\rho S^+(y) - \frac{1}{2}\{S^+(y)S^-(x), \rho\}\right)$$

Rydberg blockade and superatom

M. D. Lukin et al.,Phys. Rev. Lett. **87**, 037901 (2001)



$$|G\rangle = |g_1, \ldots, g_N\rangle \qquad S^-(x)|G\rangle = 0$$

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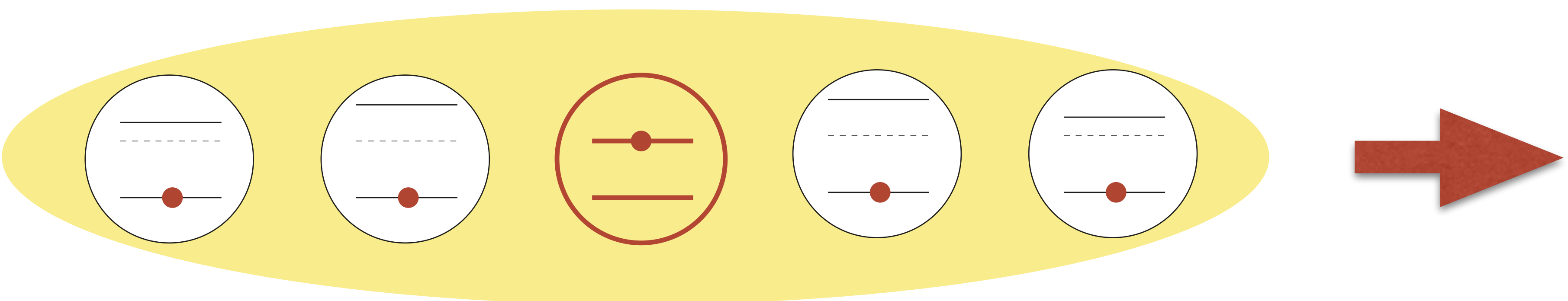

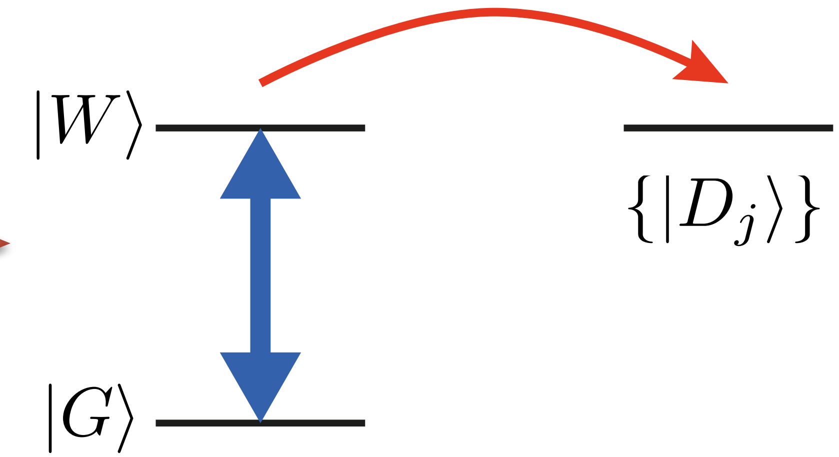

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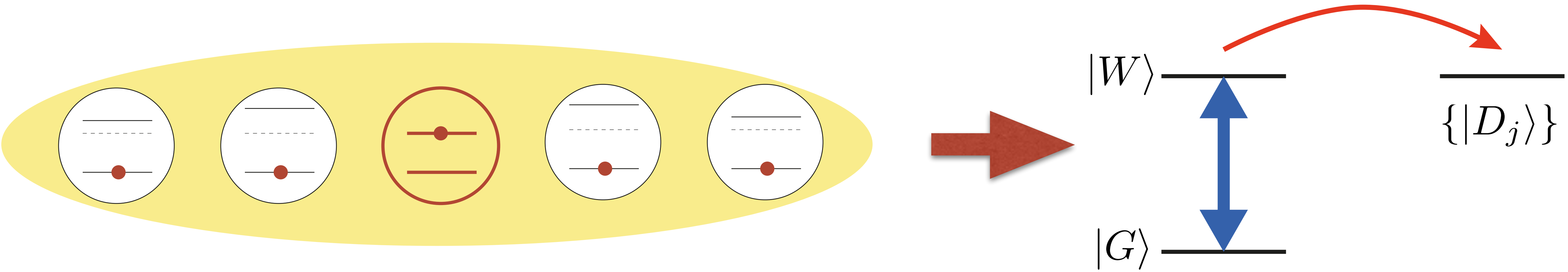

- strong dipole-dipole interaction leads to 'Rydberg' blockade
- system can be described in terms of two (collective) states and dark state manifold

'bright' state $\quad |W\rangle = \frac{1}{\sqrt{N}} \int dx\, S^{+}(x) e^{ikx} |G\rangle \quad\quad S^{+}(x)|W\rangle = 0$

'dark' states $\quad |D_j\rangle \quad\quad S^{+}(x)|D_i\rangle = 0$

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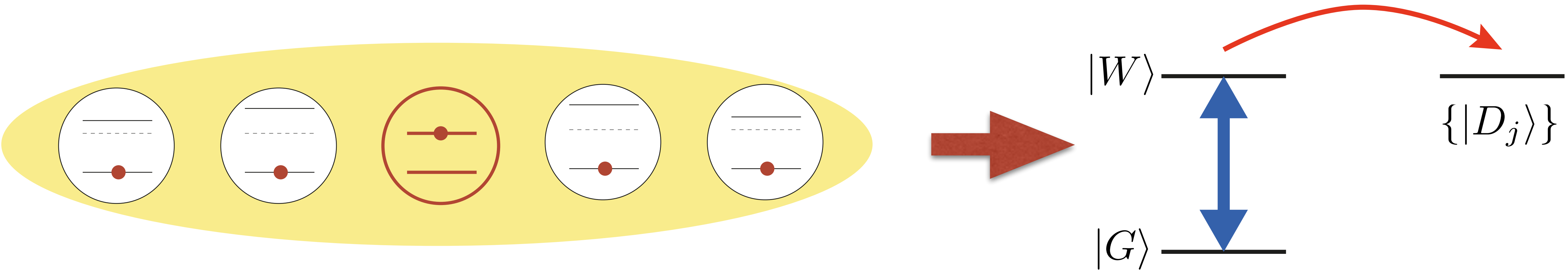

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→ interactions between atoms induce dephasing into dark state manifold

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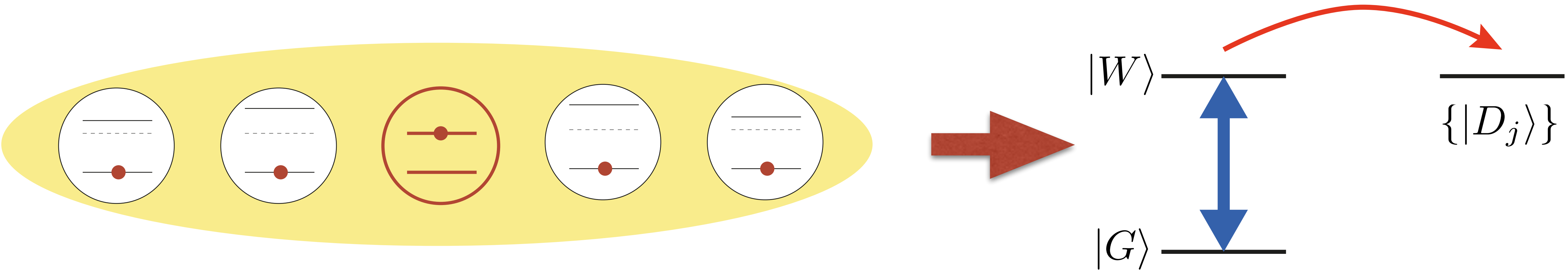

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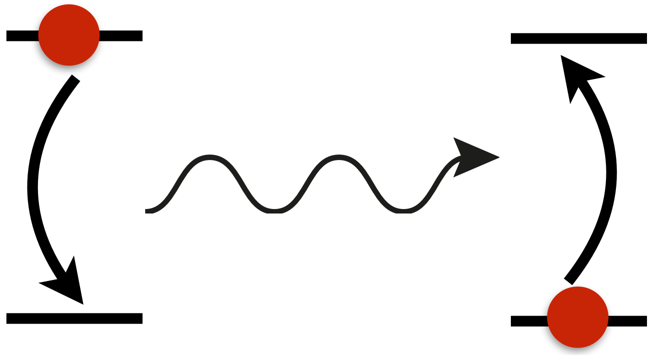
➡ interactions between atoms induce dephasing into dark state manifold

➡ gain understanding by analyzing the coherent exchange term in the master equation

Exchange interaction

$$H_D = \hbar\gamma \int dx \int dy \, \sin(k|x-y|) S^+(x) S^-(y)$$



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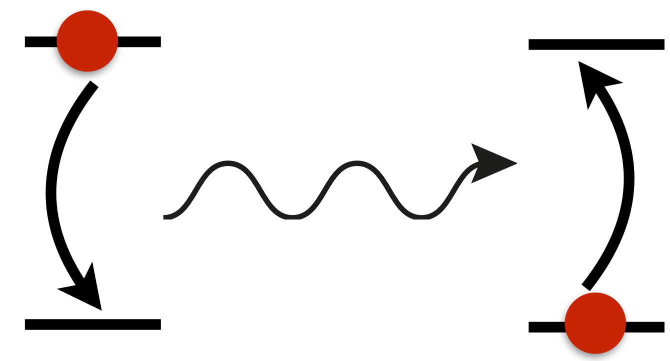

Energy shift in lowest order perturbation theory → detuning

$$\Delta = \langle W | H_D | W \rangle$$

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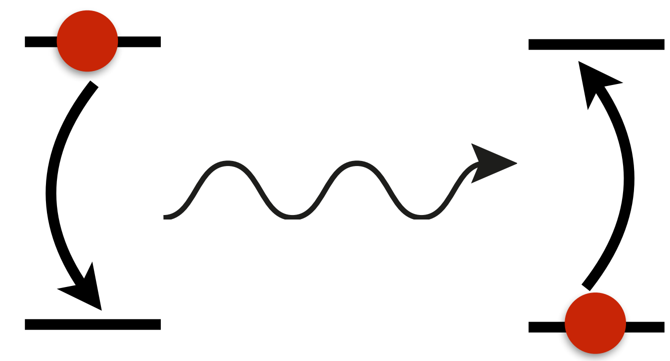
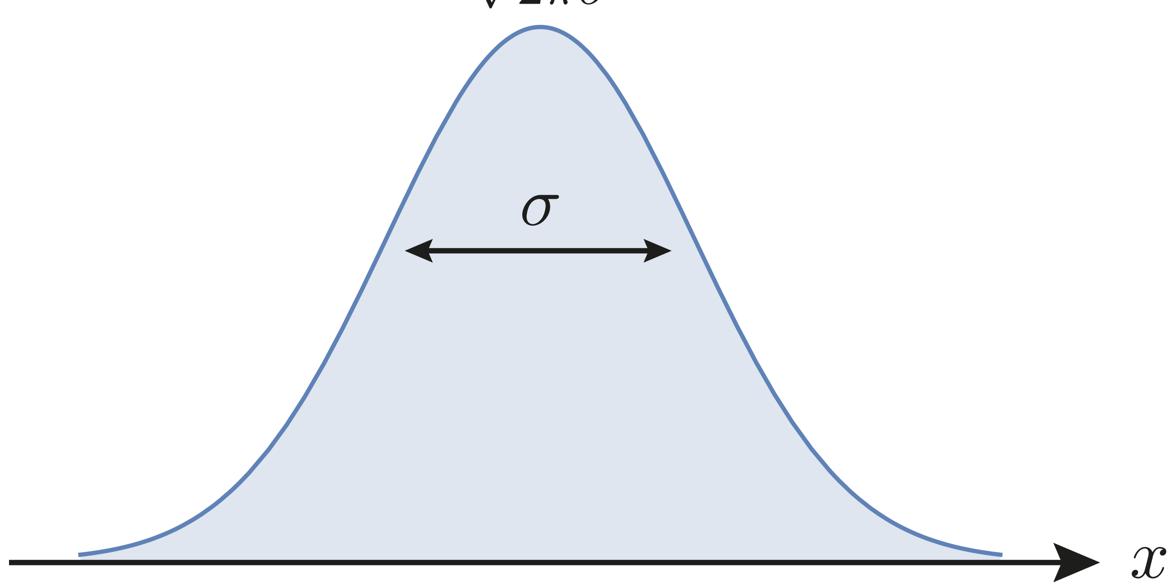


Energy shift in lowest order perturbation theory → detuning

$$\Delta = \langle W | H_D | W \rangle \xrightarrow{\langle \cdots \rangle_{\text{at}}}$$

density distribution of the atoms

$$n(x) = \frac{N}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2}$$



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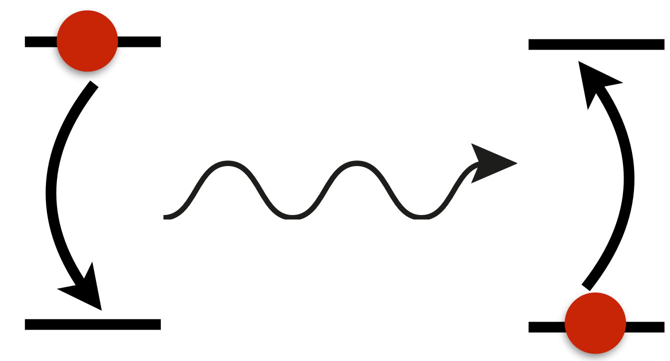

Energy shift in lowest order perturbation theory $\rightarrow$ detuning

$$\Delta = \langle W | H_D | W \rangle \xrightarrow{\langle \cdots \rangle_{\mathrm{at}}} \overline{\Delta} = \frac{(N-1)\hbar\gamma}{\sqrt{\pi}} D(2k\sigma) \sim \frac{1}{k\sigma}, \quad k\sigma \gg 1$$

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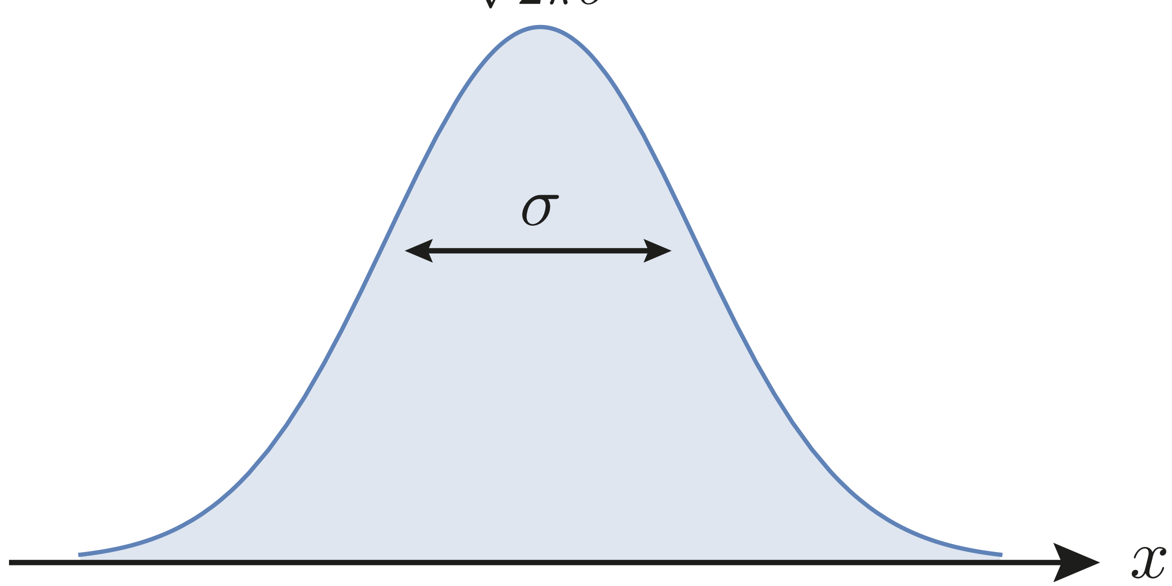

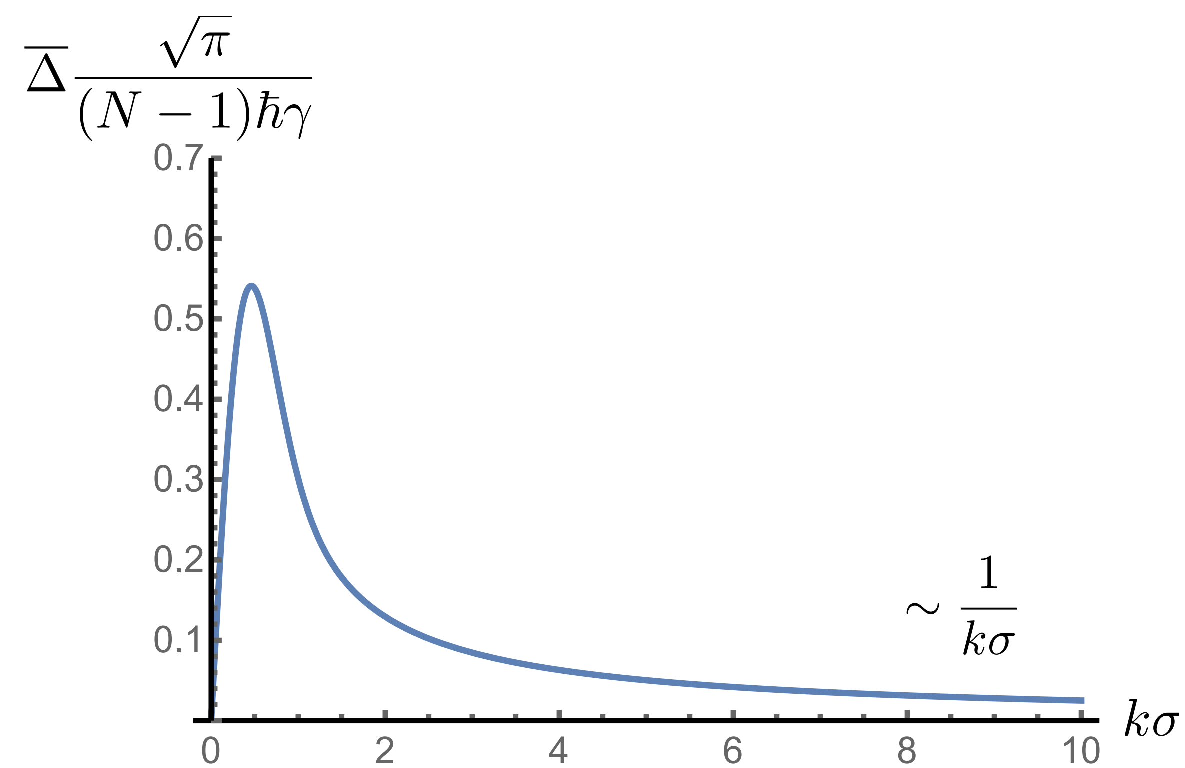

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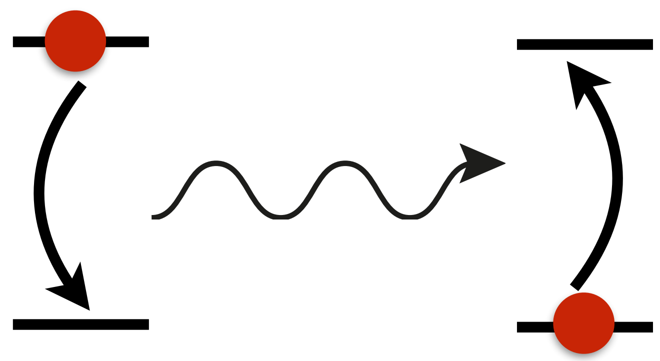

probability to find system in $|W\rangle$ state

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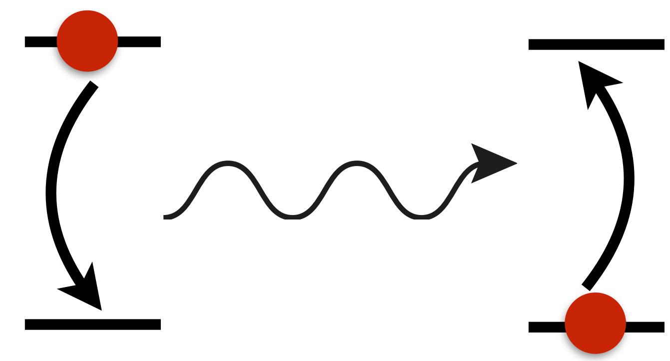

probability to find system in $|W\rangle$ state

$$p_W(t) = |\langle W|e^{-iH_D t/\hbar^2}|W\rangle \approx e^{-(\Delta H_D)^2 t^2/\hbar^2} = e^{-t^2/\tau_D^2}$$

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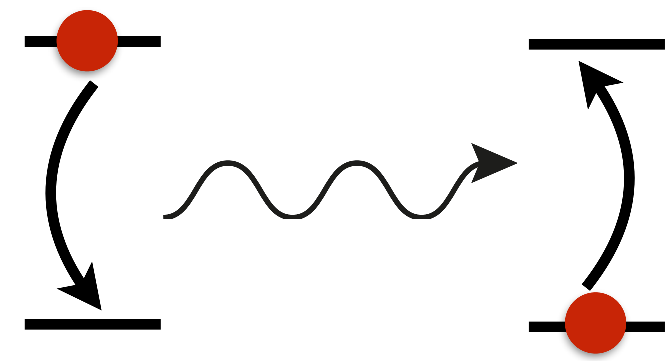

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$k\sigma \gg 1$ ➡ $\tau_D = \hbar\gamma_D^{-1} = \dfrac{1}{\gamma\sqrt{N^2/12 + N/4 - 1/3}} \approx \dfrac{2\sqrt{3}}{N\gamma}$

➡ coherent interaction term gives rise to some sort of dephasing

Dissipative terms

$$\mathcal{D}\rho = 2\gamma \int dx \int dy \cos(k|x-y|) \left(S^-(x)\rho S^+(y) - \frac{1}{2}\{S^+(y)S^-(x), \rho\} \right)$$

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decay rate into the ground state

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decay rate into the ground state

$\rho(0) = |W\rangle\langle W|$

enhanced coupling

$$\overline{\Gamma} = \mathrm{Tr}\left\{ |G\rangle\langle G|\mathcal{D}\rho(0) \right\} = \gamma \left(1 - e^{-2(k\sigma)^2} + N(1 + e^{-2(k\sigma)^2}) \right)$$

backscattering

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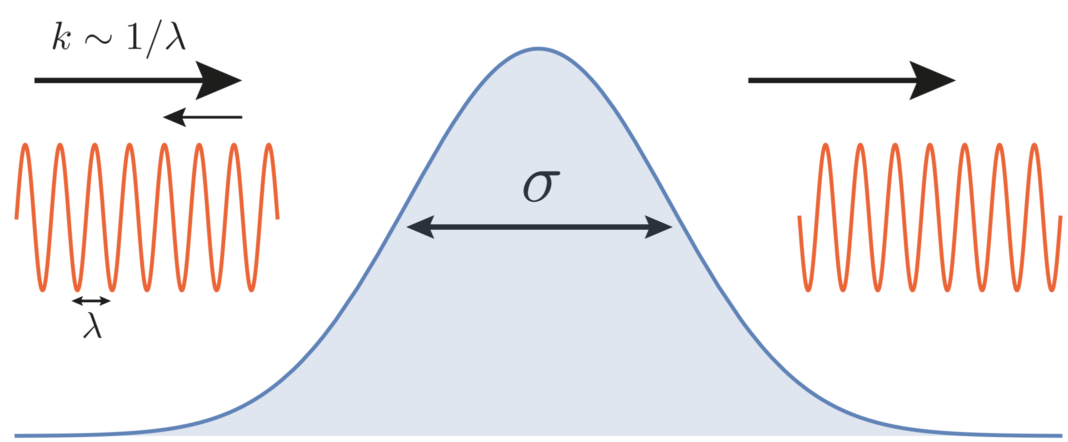
backscattering

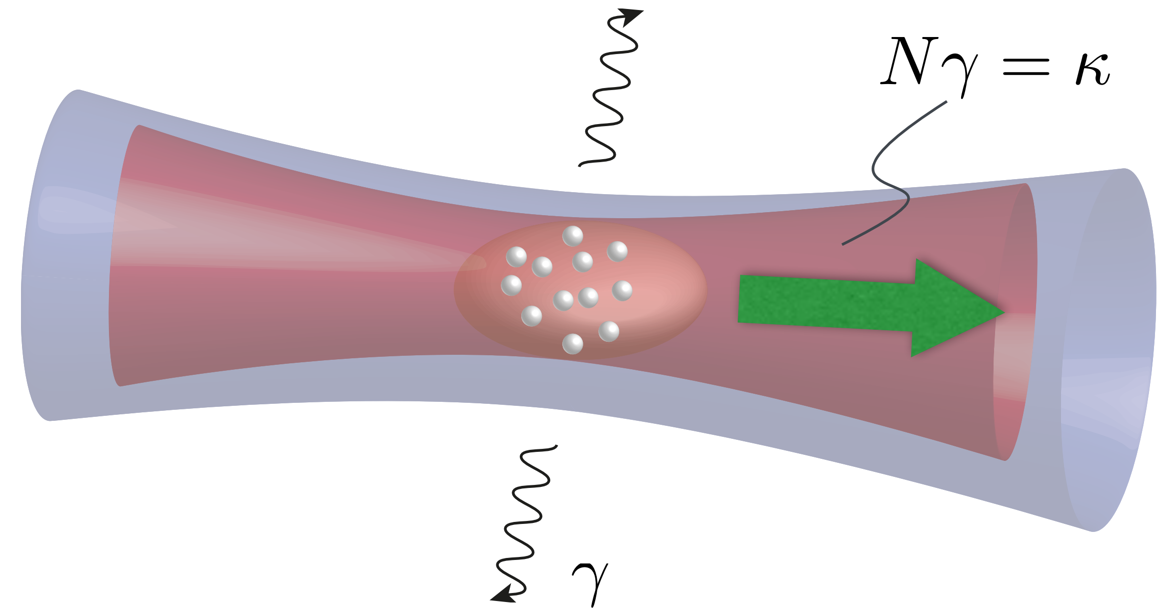
- collectively enhanced coupling to forward propagating mode
- backscattering can be neglected for $k\sigma \gg 1$



Master equation

Master equation

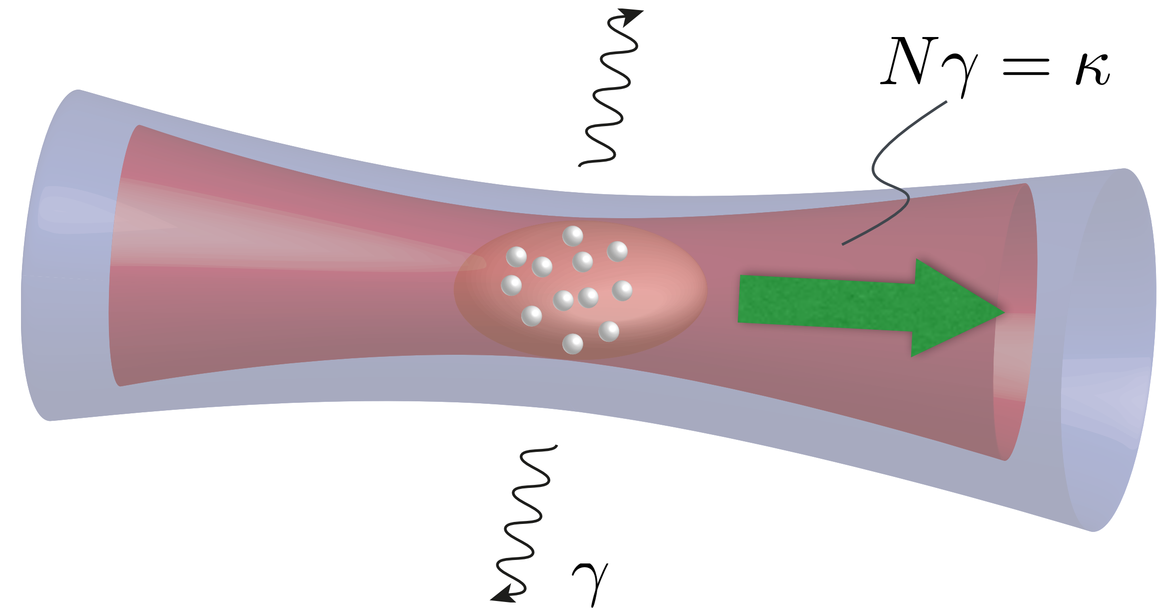
- enhanced coupling to forward propagating mode
- dephasing due to exchange interactions



$$H_{\text{sys}} = \hbar\sqrt{\gamma\, n_{\text{ph}}} \int dx\, e^{ikx}\; S^{+}(x) \,+\, \text{h.c}$$
$$\to \hbar\sqrt{\kappa\, n_{\text{ph}}}\, |W\rangle\langle G| \,+\, \text{h.c.}$$

Master equation

- enhanced coupling to forward propagating mode
- dephasing due to exchange interactions



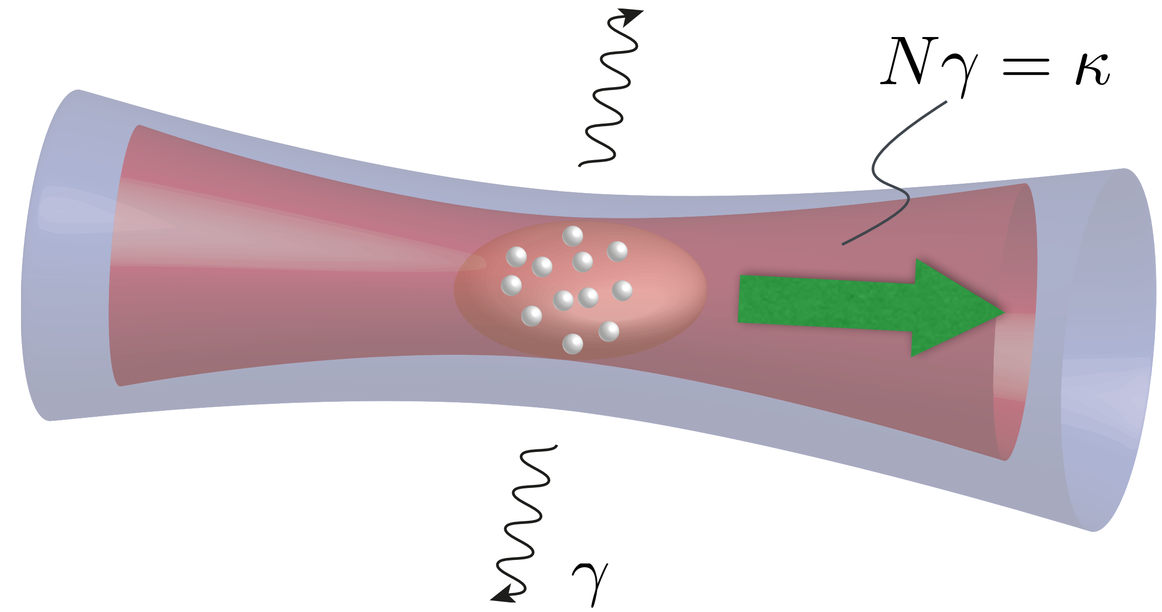
$$H_{\mathrm{sys}} = \hbar\sqrt{\gamma\, n_{\mathrm{ph}}} \int dx\, e^{ikx}\; S^{+}(x) + \mathrm{h.c}$$

$$\rightarrow \hbar\sqrt{\kappa\, n_{\mathrm{ph}}}\, |W\rangle\langle G| + \mathrm{h.c.}$$

3D analysis: decay into transverse modes with rate γ

Master equation

- enhanced coupling to forward propagating mode
- dephasing due to exchange interactions



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3D analysis: decay into transverse modes with rate γ

$$\sigma_{\alpha\beta} = |\alpha\rangle\langle\beta|$$

$$\boxed{\partial_t \rho = -\frac{i}{\hbar}\left[H_{\mathrm{sys}}(t), \rho\right] + (\kappa+\gamma)\mathcal{D}[\sigma_{GW}]\rho + \gamma_D \mathcal{D}[\sigma_{DW}]\rho + \gamma\mathcal{D}[\sigma_{DG}]\rho}$$

$$H_{\mathrm{sys}}(t) = \hbar\sqrt{\kappa}\left(\alpha^*(t)\sigma_{\mathrm{GW}} + \alpha(t)\sigma^{\dagger}_{\mathrm{GW}}\right) \qquad \mathcal{D}[\sigma]\rho = \sigma\rho\sigma^{\dagger} - \frac{1}{2}\left\{\sigma^{\dagger}\sigma, \rho\right\}$$

shape of incoming pulse $|\alpha(t)|^2 = n_{\mathrm{ph}}(t)$ *dissipator*

Master equation

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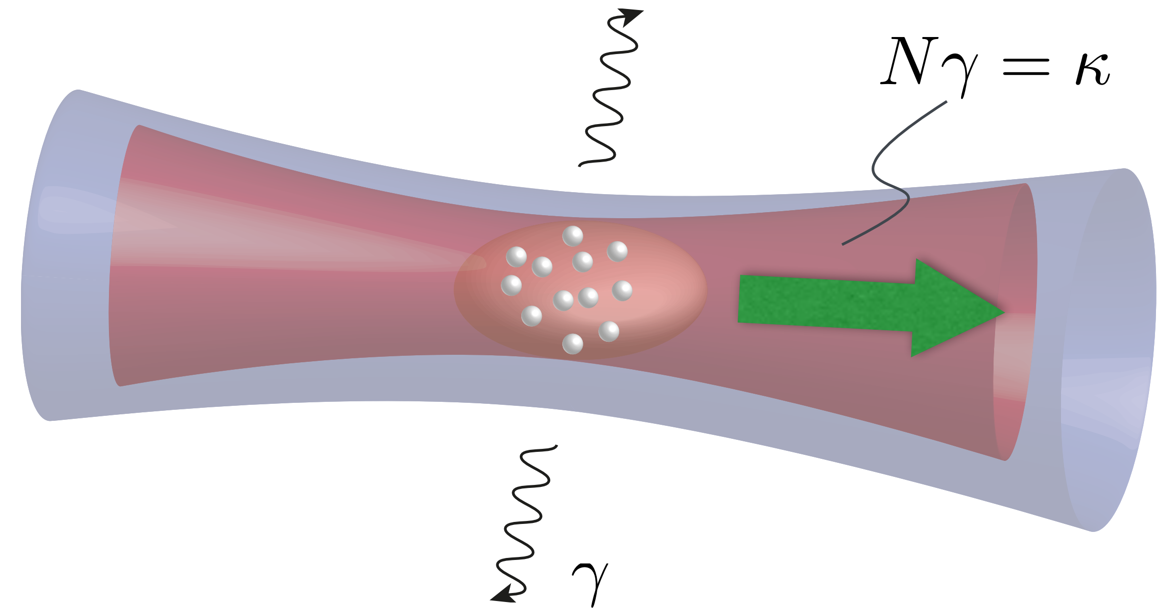

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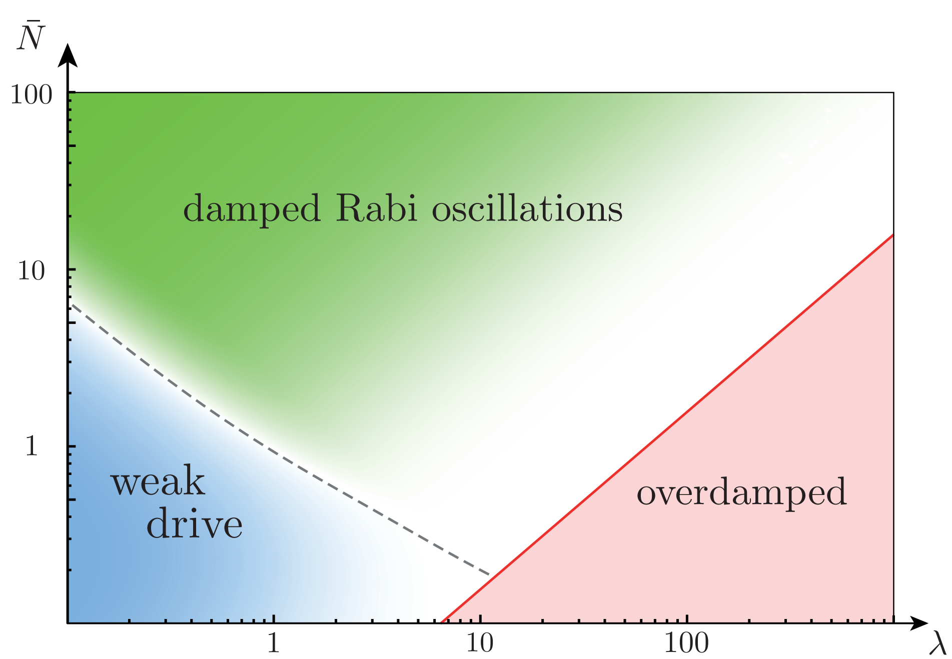
intrinsically damped system

Outline

1. Theory and model

$$\partial_t \rho = -\frac{i}{\hbar}[H,\rho] + \gamma \mathcal{D}\rho$$



2. Dynamical phase diagram

Dynamical phase diagram

Ideal case: no dephasing, no decay

$$\alpha(t) = \alpha \quad \longrightarrow \quad \rho_{\mathrm{ww}}(t) = \frac{4\alpha\kappa}{\kappa^2 + 8\alpha\kappa}\left(1 - e^{-\frac{3}{4}\kappa t}\left(\cos\Omega_{\mathrm{eff}}t + \frac{3\kappa}{4\Omega_{\mathrm{eff}}}\sin\Omega_{\mathrm{eff}}t\right)\right)$$

$$\Omega_{\mathrm{eff}} = \sqrt{4\alpha\kappa - (\kappa/4)^2}$$

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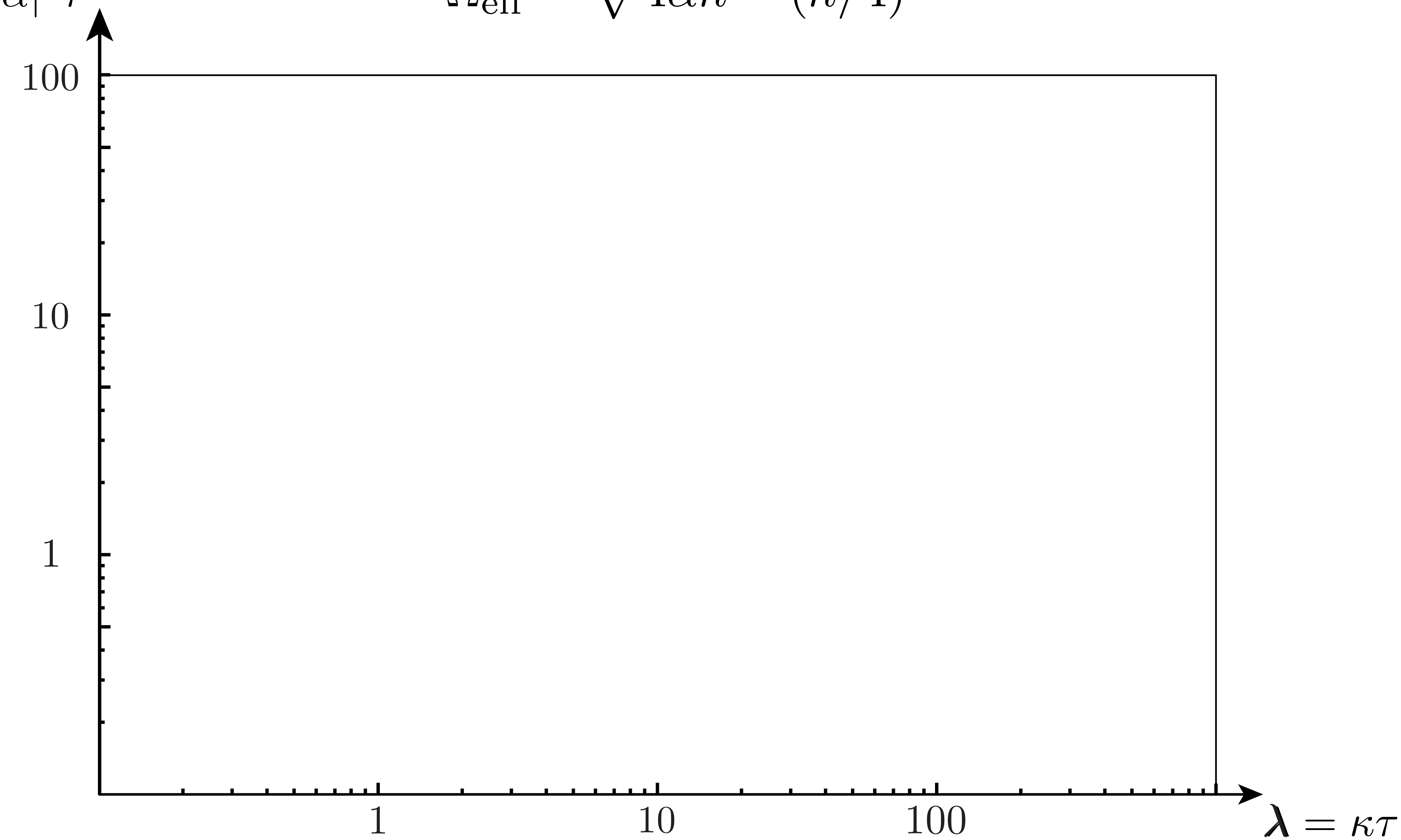
$\bar{N} = |\alpha|^2\tau$

$$\Omega_{\mathrm{eff}} = \sqrt{4\alpha\kappa - (\kappa/4)^2}$$

mean photon number



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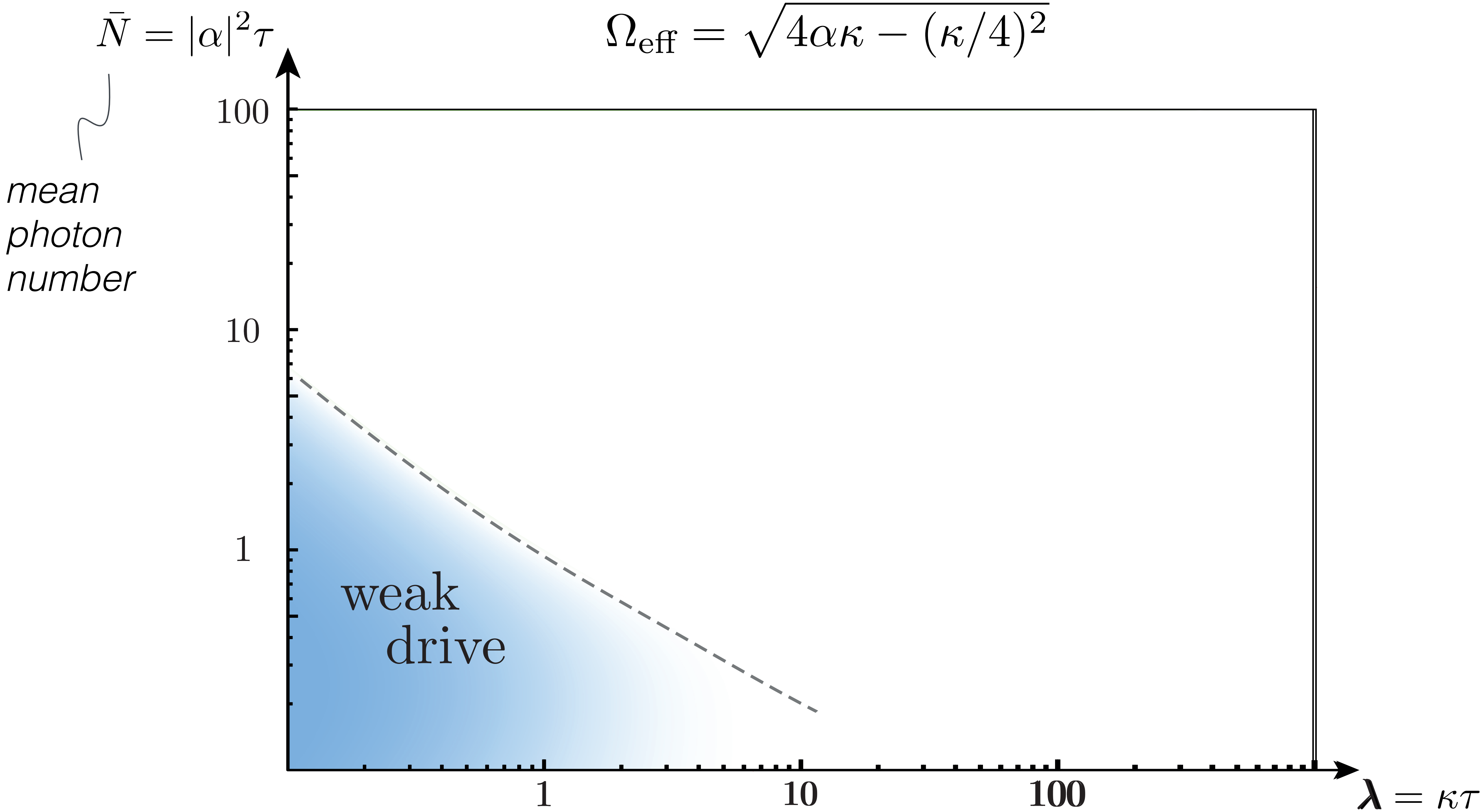

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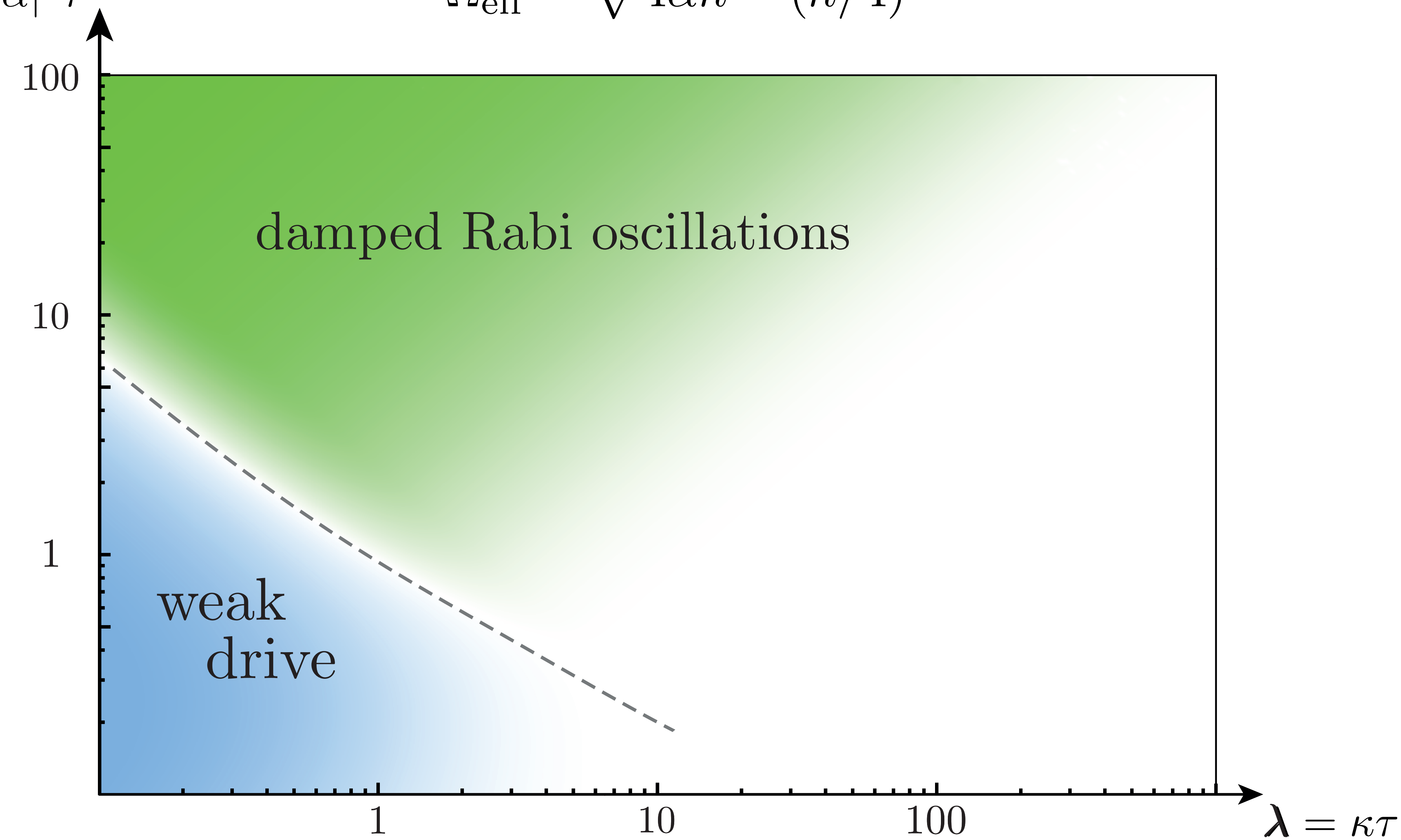

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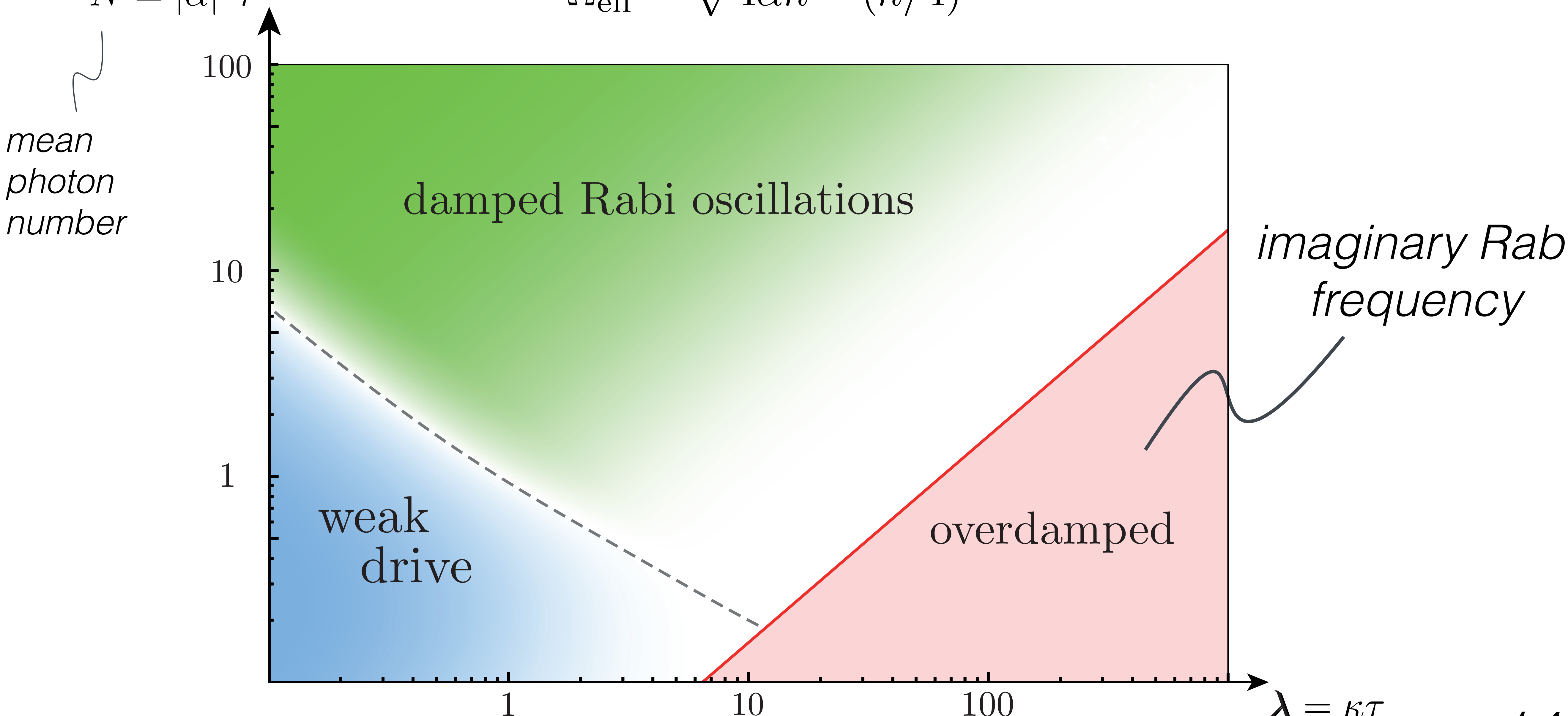
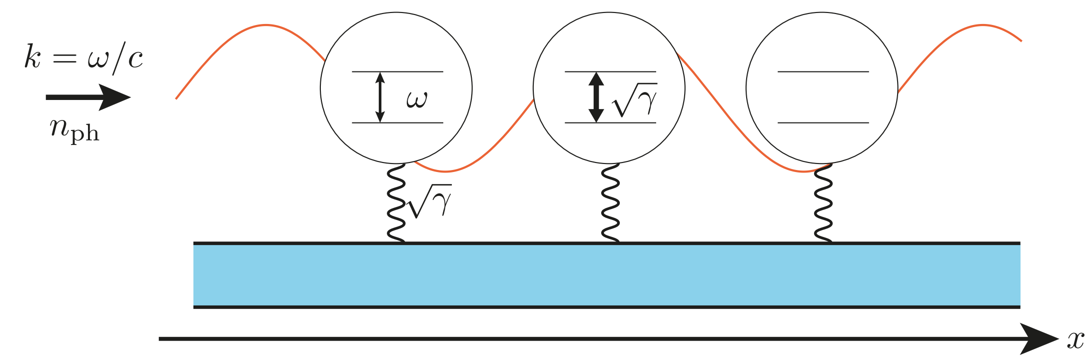
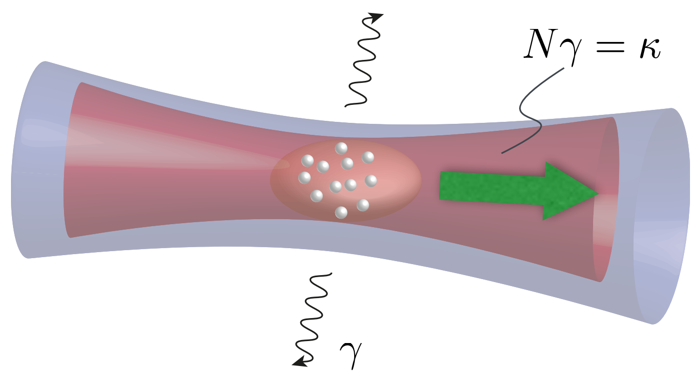


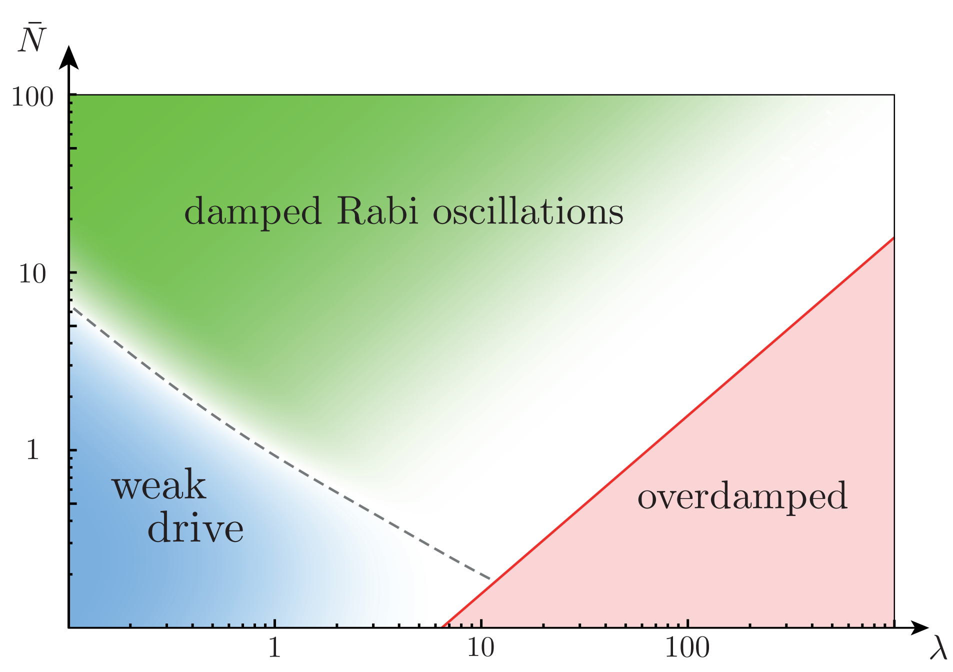
Summary

- *use 1D waveguide model for qualitative understanding of superatom*
- *enhanced coupling to forward propagating modes*
- *dephasing term due to exchange interactions*
- *microscopic derivation of superatom master equation*





$$\partial_t \rho = -\frac{i}{\hbar}[H, \rho] + \gamma \mathcal{D} \rho$$



- *even few photons can drive Rabi oscillations*
- *no Rabi oscillations for very strong coupling*

Photon correlations

Second-order correlation function (intensity correlations)

$$g^{(2)}(t_1,t_2) = \frac{\langle E^\dagger(t_1)E^\dagger(t_2)E(t_2)E(t_1)\rangle}{\langle E^\dagger(t_1)E(t_1)\rangle\langle E^\dagger(t_2)E(t_2)\rangle}$$

Reminder: $$E(t) = \alpha(t) - i\sqrt{\kappa}\sigma_{\mathrm{GW}}(t)$$

➡ *all information in atomic operators*

How to evaluate multitime correlators?

➡ *Quantum Regression Theorem*

Quantum Regression Theorem

Time evolution of density matrix $\partial_t \rho(t) = \mathcal{L}(t)\rho(t)$

$$\rho(t) = \mathcal{V}(t, t_0)\rho(t_0) = \mathcal{T}\exp\left(\int_{t_0}^{t} dt'\, \mathcal{L}(t')\right)\rho(t_0)$$

propagator for the density matrix

Quantum Regression Theorem

M. Lax, Phys. Rev. **129**, 2342 (1963)
C. Gardiner, P. Zoller, Quantum Noise (2004)

$$\langle A(t+\tau)B(t)\rangle = \mathrm{Tr}_S\left\{A\mathcal{V}(t+\tau, t)\left[B\rho(t)\right]\right\}$$

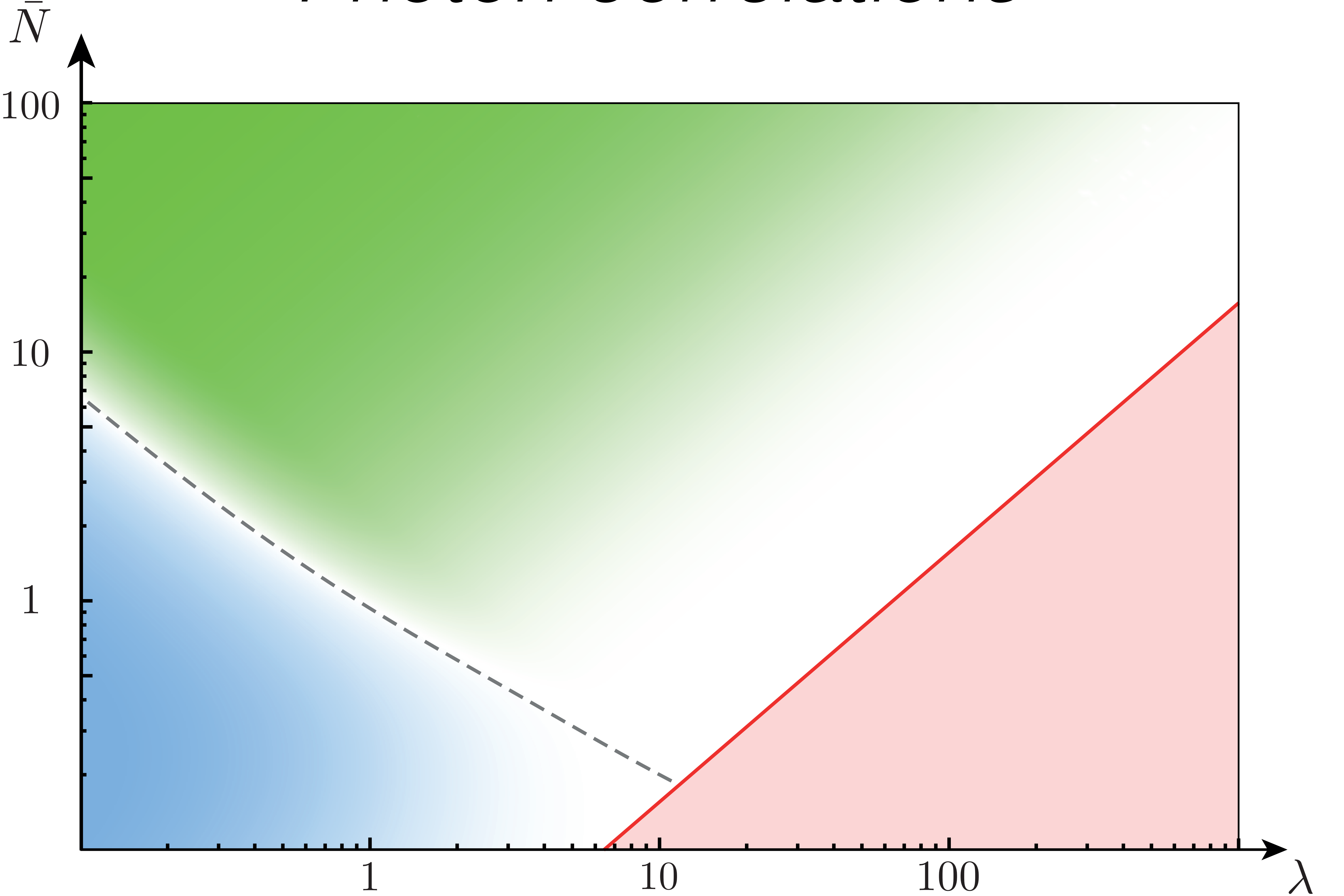
$$\langle A(t_1)B(t_2)C(t_2)D(t_1)\rangle = \mathrm{Tr}_S\left\{BC\mathcal{V}(t_2, t_1)\left[D\rho(t_1)A\right]\right\}, \quad t_2 > t_1$$

requires Born (-Markov) approximation!

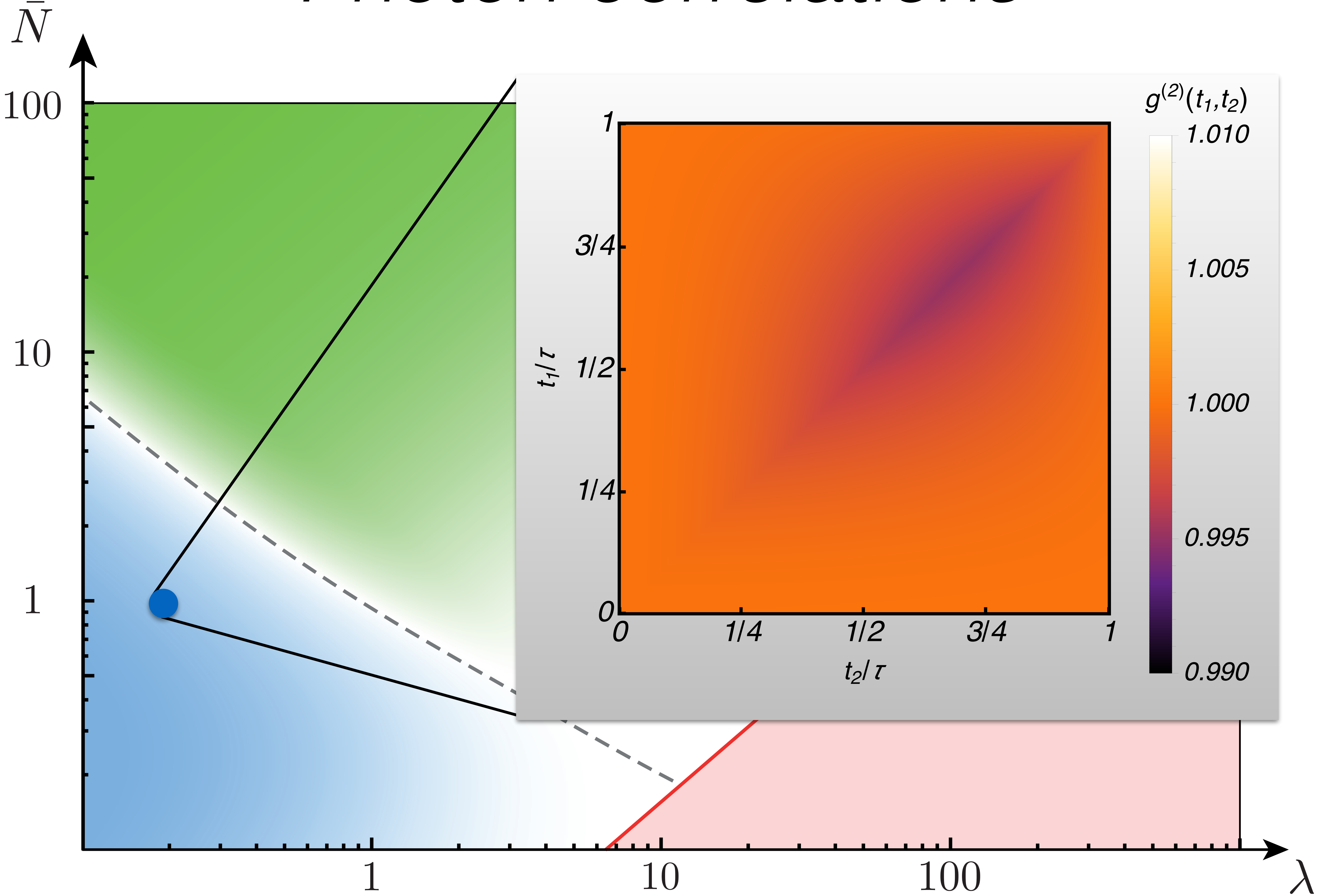
Quantum Regression Theorem exact for single two-level emitter

*T. Shi et al., PRA **92**, 053834 (2015)*

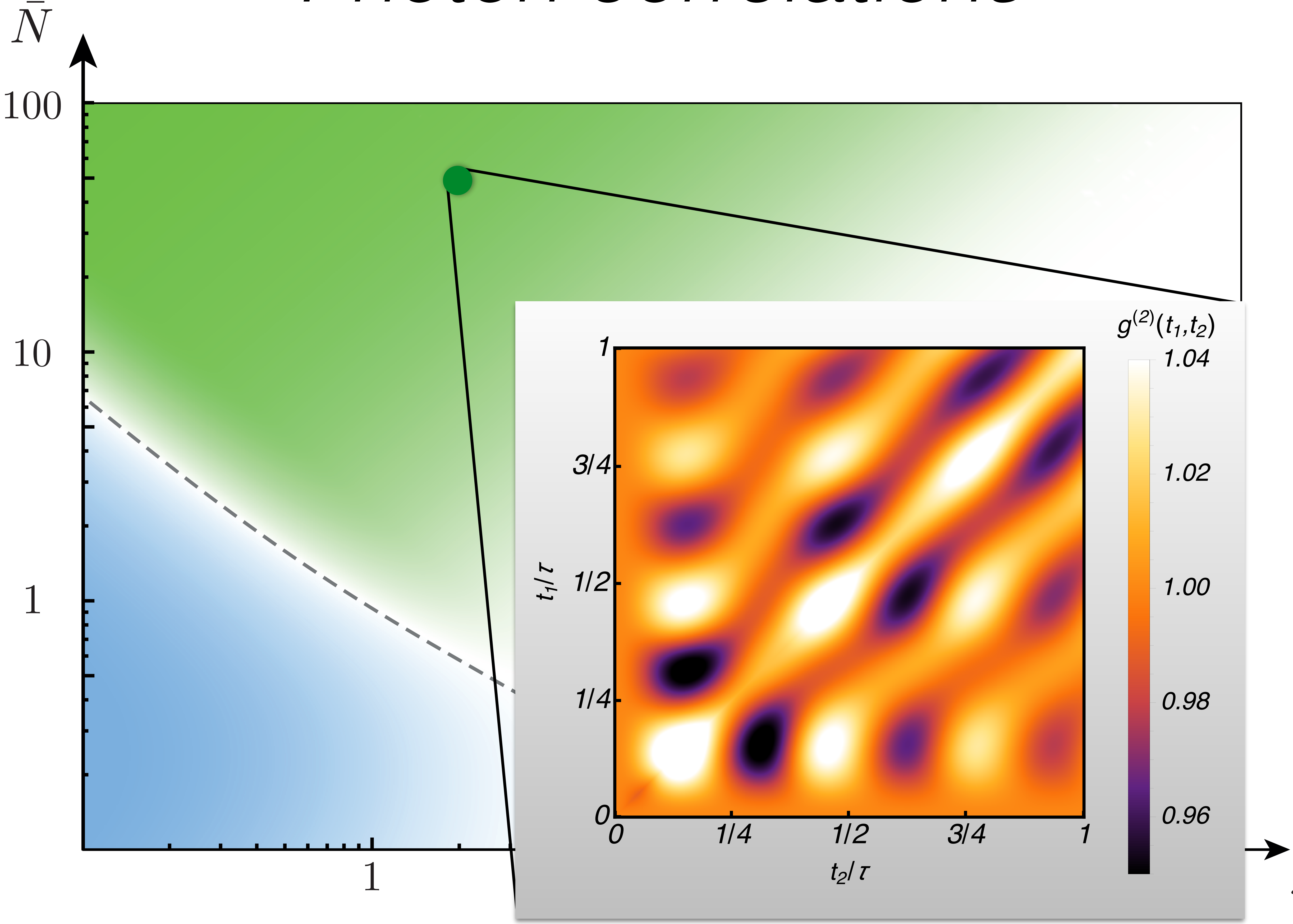
Photon correlations



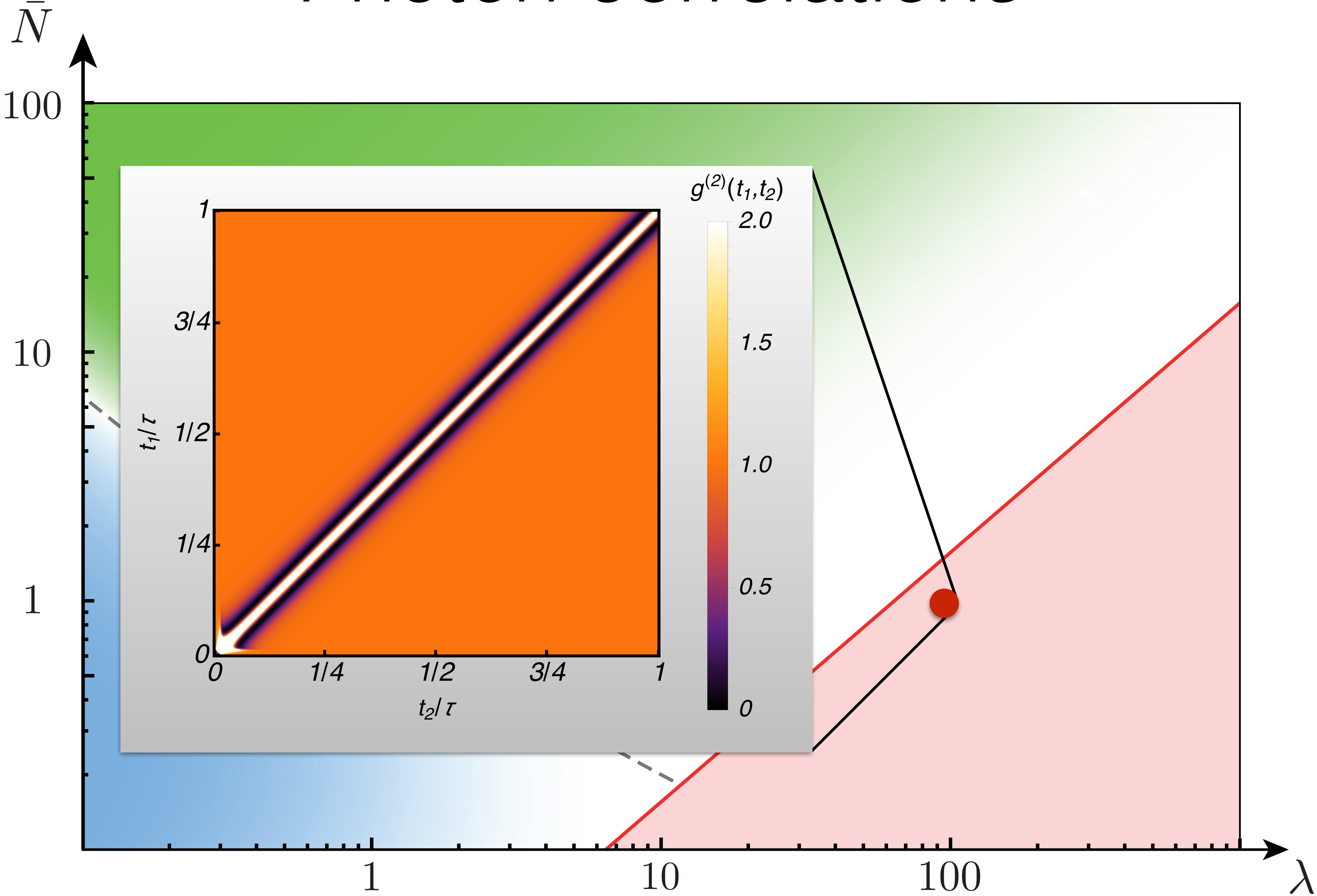
Photon correlations



Photon correlations



Photon correlations



Photon correlations