Probing Quantum Phases by Driven Dissipation

To be published.

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Abstract

We zero in on the so far rather understudied aspect of **dissipatively driven quantum simulation**, that is, the utilization of purely dissipative interactions to explore quantum phases and non-equilibrium phase transitions. To elucidate these concepts, we scrutinise exemplarily **two lattice theories of spins** coupled to tailored Markovian baths in the **absence of any unitary dynamics**, namely the paradigmatic **transverse field Ising model** and a considerably more complex $\mathbb{Z}_2$ **lattice gauge theory** with coupled Higgs field. We show that pure representatives of the quantum phases can be realized in limiting cases and illustrate that the non-equilibrium mean field phase diagrams parallel the quantum phase diagrams of the Hamiltonian "blue print" theories qualitatively.

1 Open Quantum Systems

+ The time evolution of Markovian open quantum systems can be described by a **Lindblad master equation [1]**:

$$\dot{\rho} = \sum_i \left[L_i \rho L_i^\dagger - \frac{1}{2}\left\{L_i^\dagger L_i, \rho\right\}\right] \equiv \mathcal{L}\rho$$

(ρ density matrix, L_i non-hermitian jump operators)

+ The jump operators can either be derived microscopically or contrived by hand (quantum simulator). *We do the latter.*

+ **Generic solution:** $\rho(t) = \exp(\mathcal{L}t)\rho_0$ with initial state ρ_0.

+ Fixed points $\mathcal{L}\rho = 0$ are termed **non-equilibrium steady states (NESS)**. The pure ones with $L_i\rho = 0$ are called **dark states**.

2 The Dissipative Transverse Ising Model

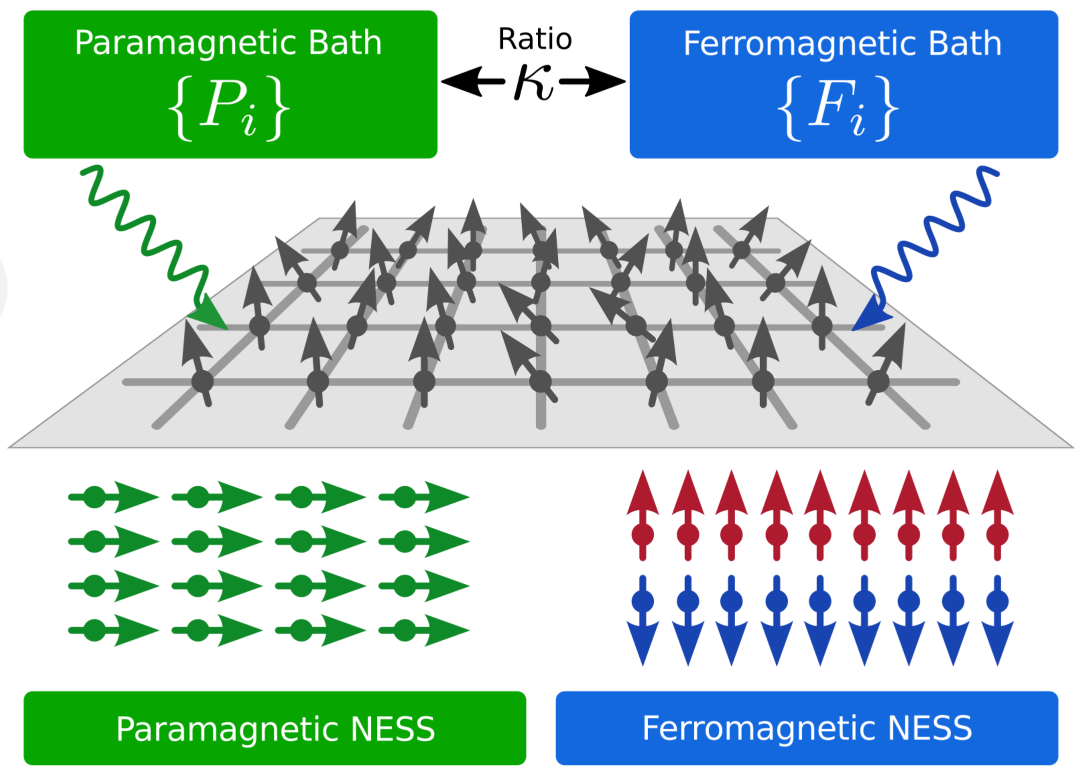
+ We aim at a purely dissipative analogue of the paradigmatic **transverse Ising model (TIM)** $H_{\text{TIM}} = -\sum_{\langle s,t\rangle} \sigma_s^z\sigma_t^z - g\sum_s \sigma_s^x$ with unique ground state $|+\cdots+\rangle$ for $g=\infty$ and degenerate ground states $\{|\uparrow\ldots\uparrow\rangle, |\downarrow\ldots\downarrow\rangle\}$ for $g=0$ [2].

+ **Introduce two competing jump operators for each site of a rectangular lattice:**

$$P_s \equiv \sqrt{\kappa}\,\sigma_s^z\left[\mathbb{1} - \sigma_s^x\right] \quad \text{and} \quad F_s \equiv \sigma_s^x\left[\mathbb{1} - \frac{1}{q}\sum_{t\in s}\sigma_t^z\sigma_s^z\right]$$

(κ relative bath strength, σ_s Pauli matrix on site s, q coordination number, $t\in s$ neighbours of s)

+ **Dark states** for $\kappa = 0|\infty$ correspond to TIM ground states for $g = 0|\infty$ (ferromagnetic | paramagnetic).



3 Results for the dissipative Transverse Ising Model

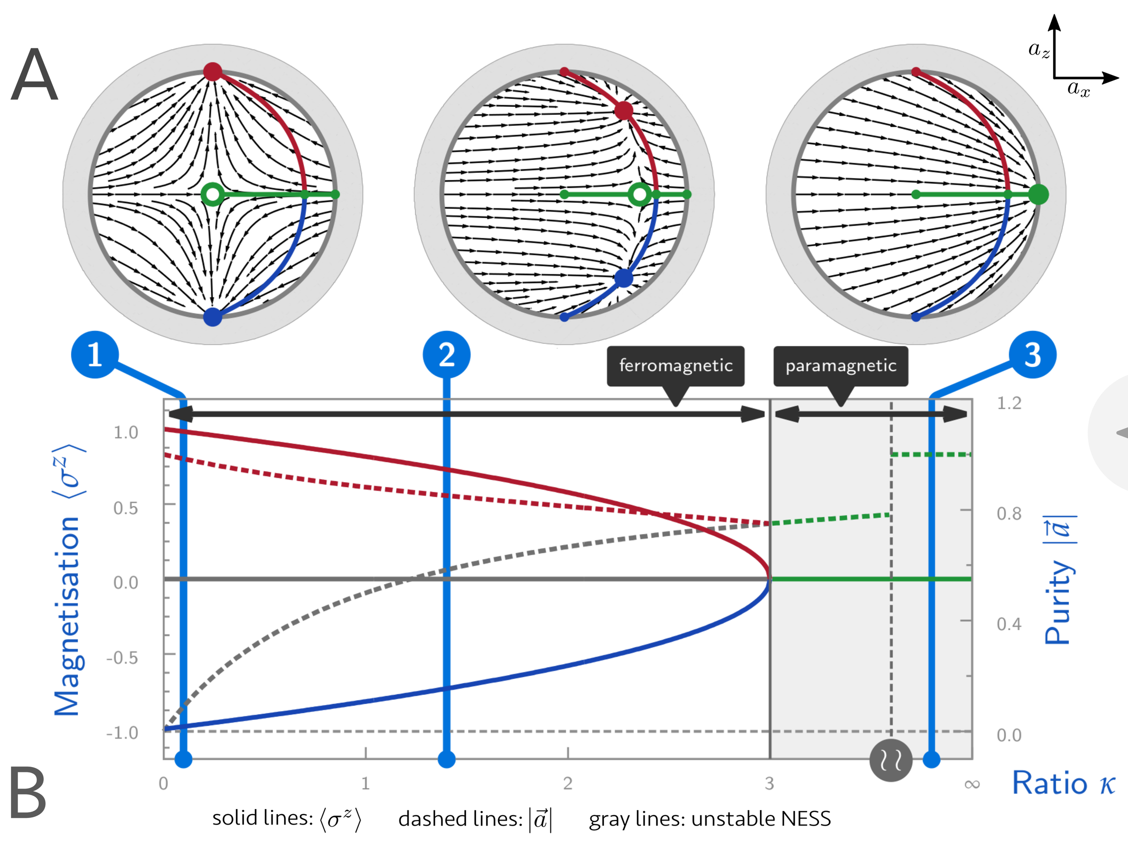
+ **What happens for** $0 < \kappa < \infty$ **?** Phase transition (PT) analogous to the TIM?

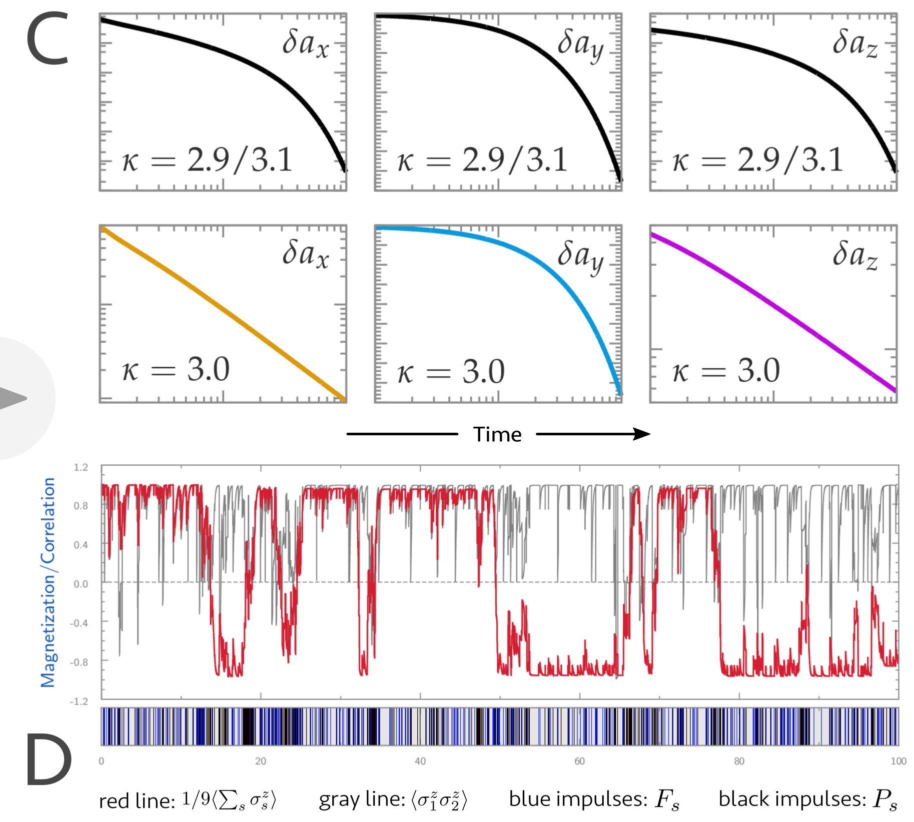
+ **Mean field (MF) theory:** Product ansatz $\rho = \bigotimes_s \rho_s$ → Trace out all but one spin → Effective Lindblad operator $\mathcal{L}^{\text{mf}}$ → Apply self-consistency relation.

+ **MF phase diagram:** We find a continuous PT at $\kappa_c = 4(1-1/q)$ with a unique NESS for $\kappa \geq \kappa_c$ and two stable NESS for $\kappa < \kappa_c$ (**B**). Parametrizing $\rho_s = (\mathbb{1} + \vec{\sigma}\cdot\vec{a})/2$ with Bloch vector $\vec{a}$ yields cross-sections of Bloch spheres (**A**) where the flow encodes the relaxation towards the NESS.

+ **MF dynamics:** Expressed as dynamical system $\partial_t\vec{a} = \mathcal{L}^{\text{mf}}(\vec{a})$ of Bloch vector. Relaxation: $\vec{a}(t) = \vec{a}_{\text{NESS}} + \delta\vec{a}(t)$ → Algebraic decay of $\delta a_{x,z}$ at κ_c (**C**).

+ **Quantum Trajectory Monte-Carlo:** We simulated small (3×3) systems exactly. **D** shows a typical trajectory for $\kappa = 1/9$ with initial state $|\uparrow\rangle^{\otimes 9}$.
→ Intermittent total magnetization and jump rate.





4 The Dissipative $\mathbb{Z}_2$-Gauge-Higgs Model

+ Is this approach feasible for more complex theories?

+ Consider the $\mathbb{Z}_2$ lattice gauge theory with coupled matter field on a (hyper-)cubic lattice [3]:

$$H_{\mathbb{Z}_2\text{GH}} = -\sum_s \sigma_s^x - \lambda\sum_e I_e - \sum_e \tau_e^x - \omega\sum_p B_p \quad \text{with} \quad I_{e=(s,t)} \equiv \sigma_s^z\tau_e^z\sigma_t^z, \quad B_p \equiv \prod_{e\in p}\tau_e^z$$

(λ, ω positive parameters, σ_s matter/Higgs field on site s, τ_e gauge field on edge e, $e\in p$ bounding edges of face p, $e=(s,t)$ endpoints of edge)

+ **Local (gauge) symmetry:** $G_s \equiv \sigma_s^x\prod_{e:s\in e}\tau_e^x$ → **Physical states:** $G_s|\chi\rangle = |\chi\rangle$ (Gauss law)

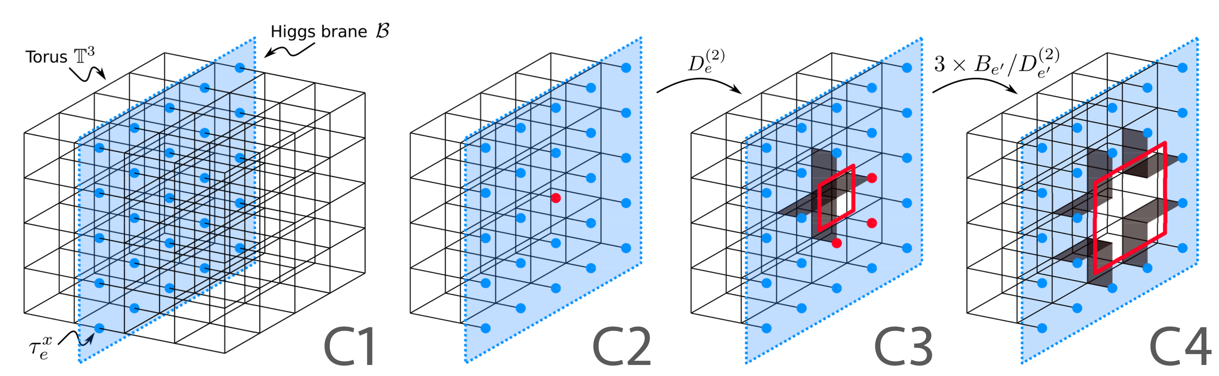
+ **Phase diagram (A) is almost completely known through analytical and numerical calculations:**
(I) confined charge phase / (II) free charge phase / (III) Higgs phase

+ **Contrive competing jump operators that drive the system in the three distinct phases:**
Generic structure: $L = \text{THEN}\cdot\text{IF}$ where IF detects elementary excitations of the desired phase, and THEN moves/annihilates them.

+ 4 terms in Hamiltonian
→ 4 jump operators?

+ **No!** Topologically non-trivial excitations cannot be contracted by local deformations.

→ Gauge string / Higgs brane **fragility** needed (**C1-4**).



+ **We propose 6 types of jump operators (B):**
(notation: $O_{x\in y} \equiv |y|^{-1}\sum_{x\in y}O_x$)

1 Gauge string tension
$F_p^{(1)} = B_p\left(\mathbb{1} - \tau_{e\in p}^x\right)$

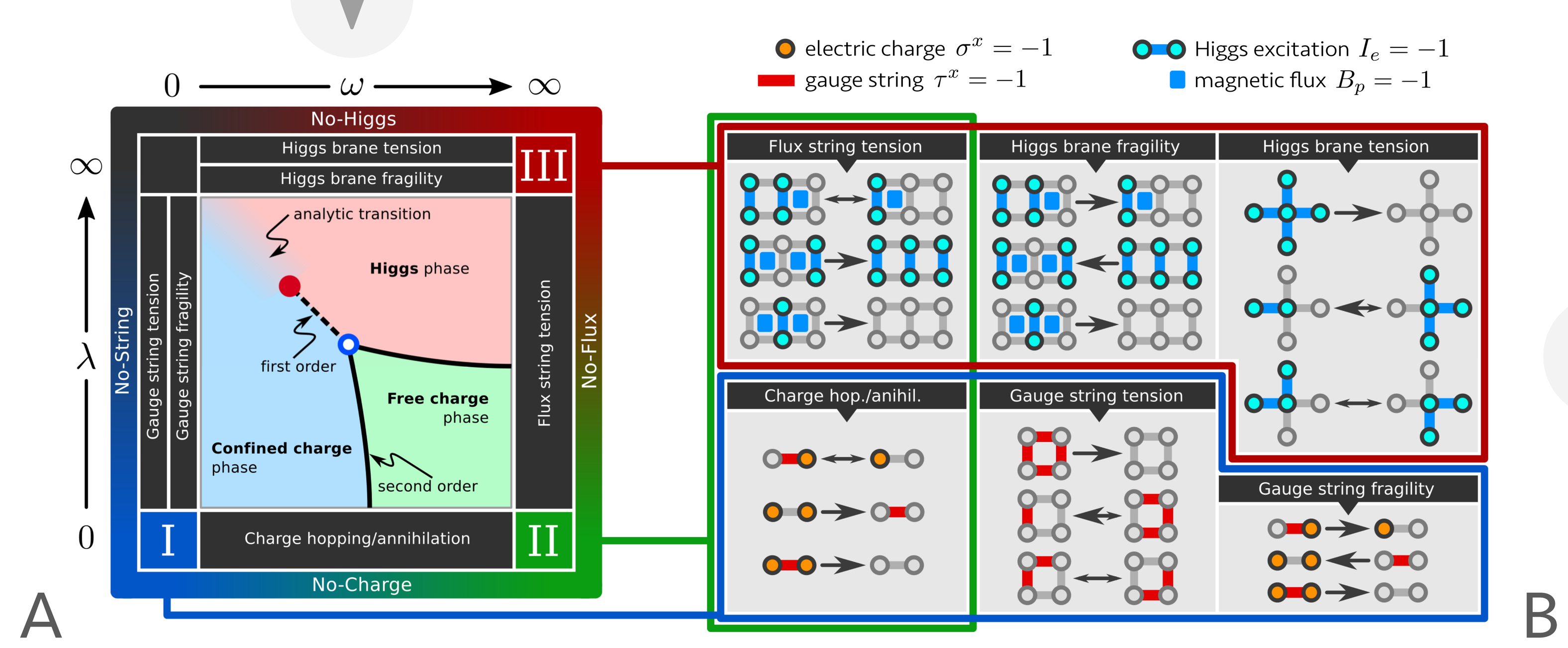
2 Gauge string fragility
$F_e^{(2)} = I_e\left(\mathbb{1} - \tau_e^x\right)$

3 Higgs brane tension
$D_s^{(1)} = \sqrt{\lambda}\,\sigma_s^x\left(\mathbb{1} - I_{e\in s}\right)$

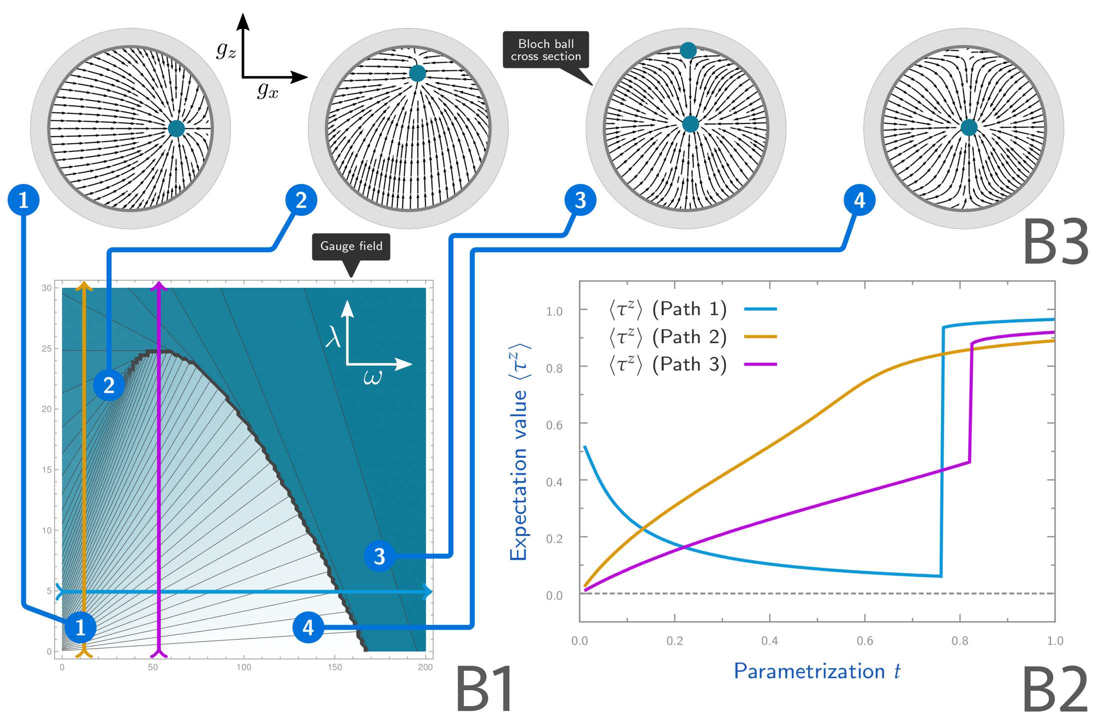
4 Higgs brane fragility
$D_e^{(2)} = \sqrt{\lambda}\,\tau_e^x\left(\mathbb{1} - I_e\right)$

5 Charge hop./annihil.
$T_e = I_e\left(\mathbb{1} - \sigma_{s\in e}^x\right)$

6 Flux string tension
$B_e = \sqrt{\omega}\,\tau_e^x\left(\mathbb{1} - B_{p\in e}\right)$



5 Results for the dissipative $\mathbb{Z}_2$-Gauge-Higgs Model



+ Fix the **unphysical degrees of freedom** by recasting the theory in **unitary gauge**:

$$T\tau_e^zT^\dagger = I_e, \quad T\tau_e^xT^\dagger = \tau_e^x, \quad T\sigma_s^zT^\dagger = \sigma_s^z, \quad T\sigma_s^xT^\dagger = G_s \;\rightarrow\; \tilde{L} = TLT^\dagger$$
: Jump operator in unitary gauge.

→ Matter field carries unphysical degrees of freedom → Fix/Drop this field.

+ **Mean field theory (B1-3):** Apply MF theory with single mean field for gauge field.
→ First order phase transition separates phases (I) and (II)/(III) + Phases (II) and (III) merge + Analytic transition between (I) and (II)/(III) is predicted.

+ **Dissipative MF phase diagram in unitary gauge parallels the well-known MF phase diagram of the Hamiltonian theory in the same gauge [4].**

+ **What happens for** $0 < \lambda < \infty$ **and** $0 < \omega < \infty$ **?** Phase transitions analogous to the Hamiltonian theory?

+ **Mean field theory:** Insert the product ansatz

$$\rho = \bigotimes_e \rho_e^{\text{g}} \otimes \bigotimes_s \rho_s^{\text{m}}$$

and proceed as in (3) with **two** separate mean fields for mass/Higgs (m) and gauge fields (g).

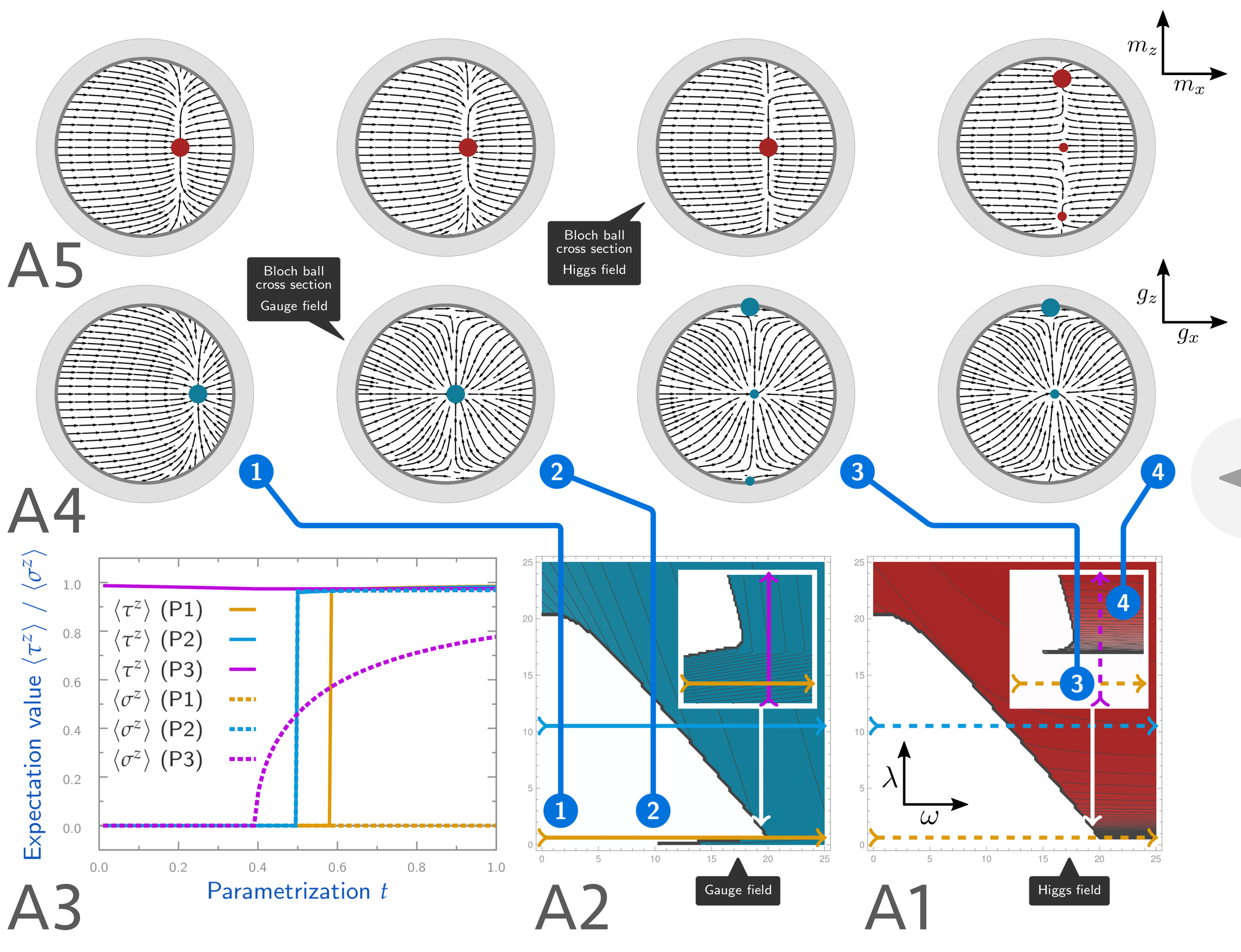
+ **MF dynamics:** There are **two** coupled Bloch vectors

$$\partial_t\vec{m} = \mathcal{L}^{\text{mf,m}}(\vec{g},\vec{m}) \quad \text{and} \quad \partial_t\vec{g} = \mathcal{L}^{\text{mf,g}}(\vec{g},\vec{m})$$

with $\rho_s^{\text{m}} = (\mathbb{1}_s + \vec{m}\vec{\sigma}_s)/2$ and $\rho_e^{\text{g}} = (\mathbb{1}_e + \vec{g}\vec{\tau}_e)/2$.

+ **Mean field theory results (A1-5):**
→ All three phases are predicted + Phases (I)/(II) and (I)/(III) separated by 1st order PT + Phases (II) and (III) separated by 2nd order PT + No analytic transition between phases (I) and (III) predicted.

+ **Dissipative MF phase diagram parallels the well-known MF phase diagram of the Hamiltonian theory [4].**



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