

David Peter

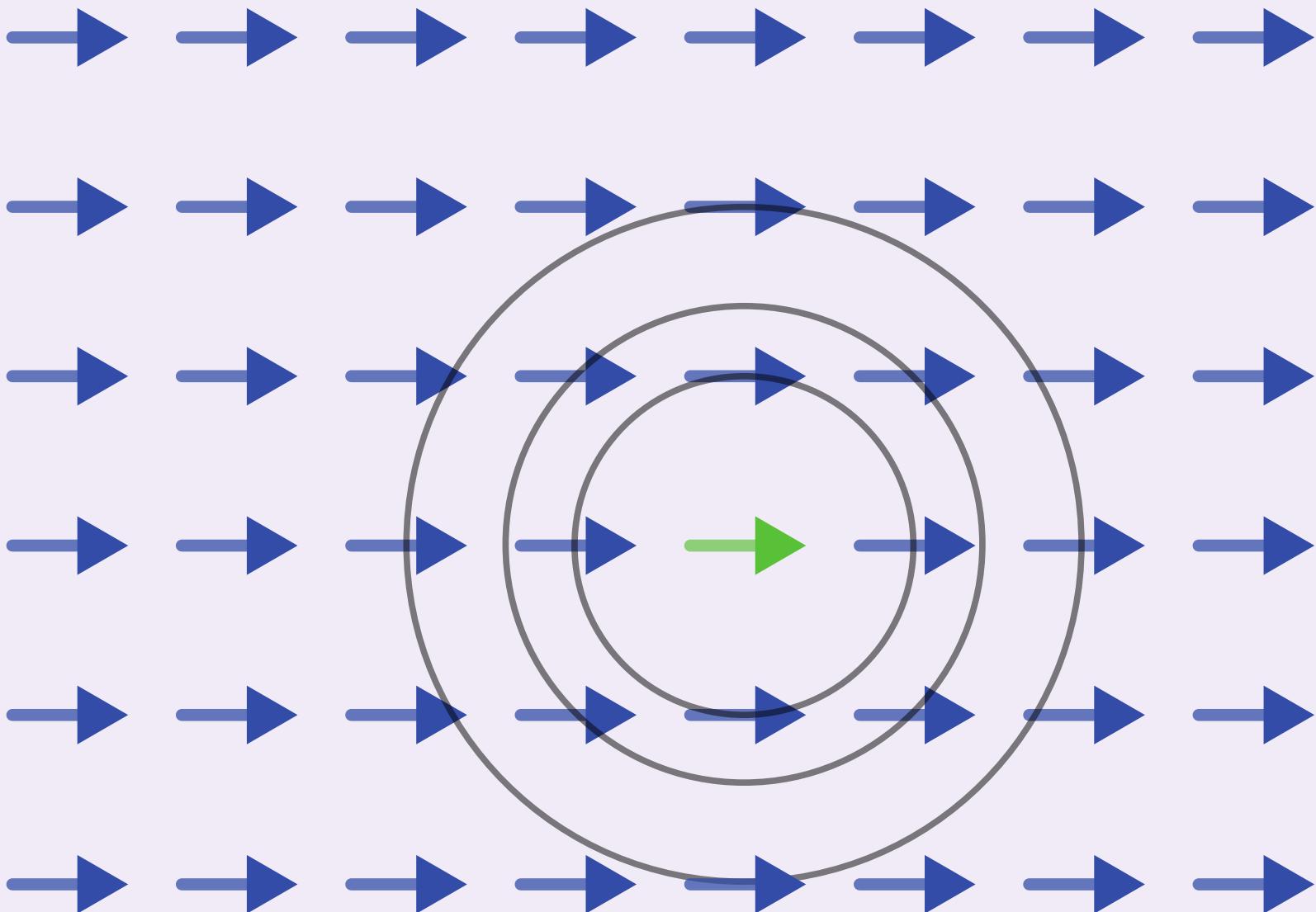
october 24th, 2012

SFB workshop, reisensburg

spin systems with dipolar interactions

B8: *quantum phase transitions with cold polar molecules*

motivation



outline

polar molecules

- ▷ realization of spin models

reminder

- ▷ 2D lattice spin models
with nearest-
neighbor interactions

dipolar spin model

- ▷ mean field prediction
- ▷ spin wave analysis

ferromagnetic Ising and XY phases

- ▷ excitations spectra
- ▷ long-range order
- ▷ spin wave dynamics

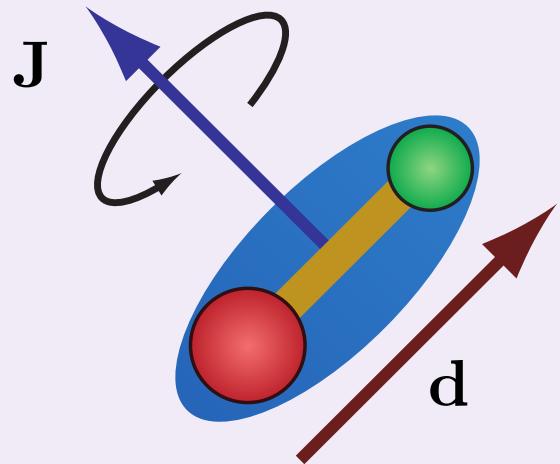
polar molecules

$$H_{\text{rot}} = B \mathbf{J}^2 - \mathbf{d} \cdot \mathbf{E}(t)$$



$$J = 2 \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad}$$

$$\begin{array}{c} J = 1 \\ \quad \quad \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \end{array} \quad \quad \quad \begin{array}{c} J = 0 \\ \quad \quad \quad \underline{\quad} \end{array} \quad \quad \quad \updownarrow \sim 20\text{GHz}$$



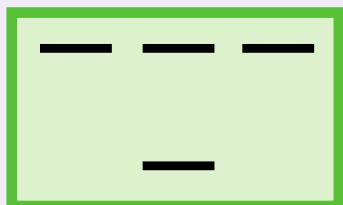
polar molecules

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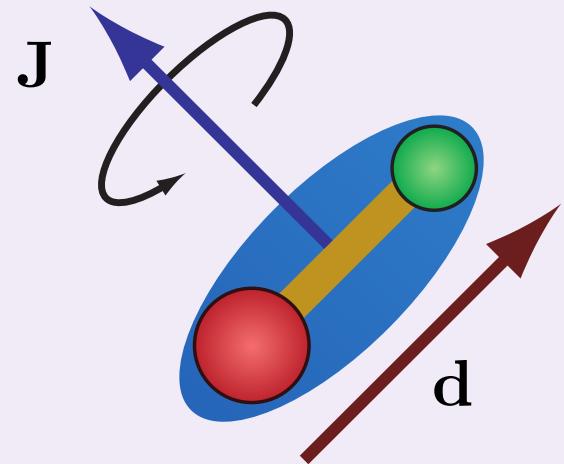


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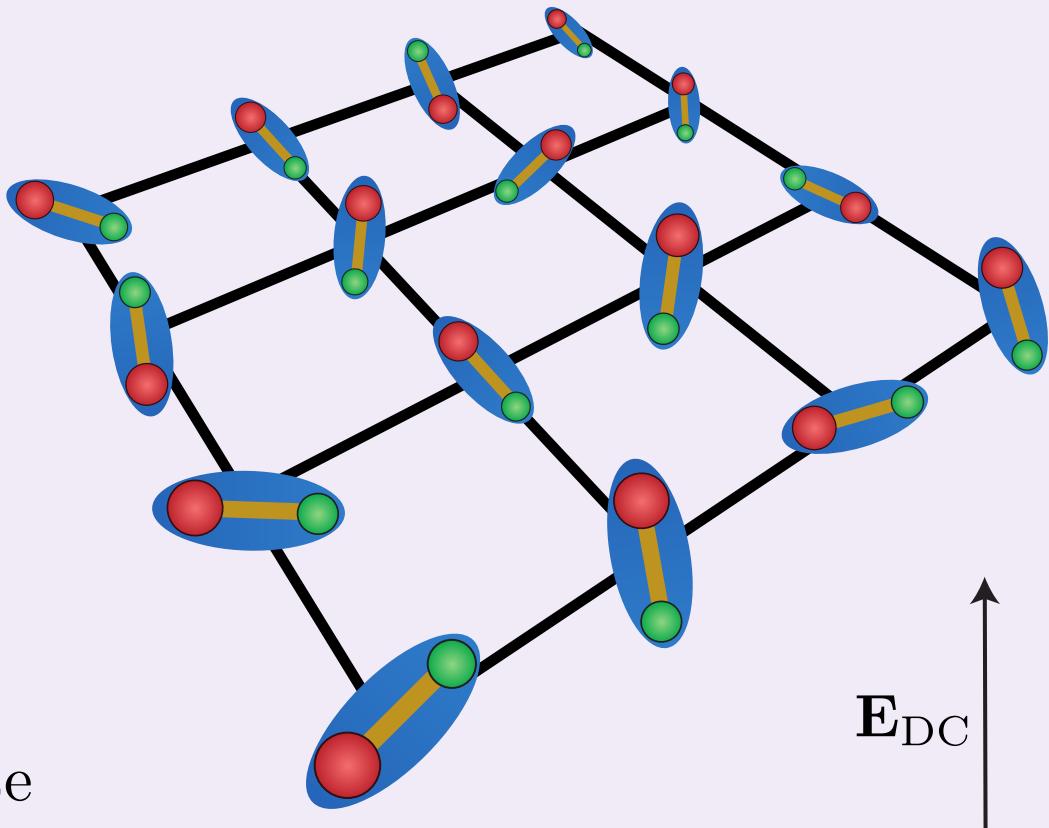


$$\mathbf{E}_{\text{DC}} = E \mathbf{e}_z$$



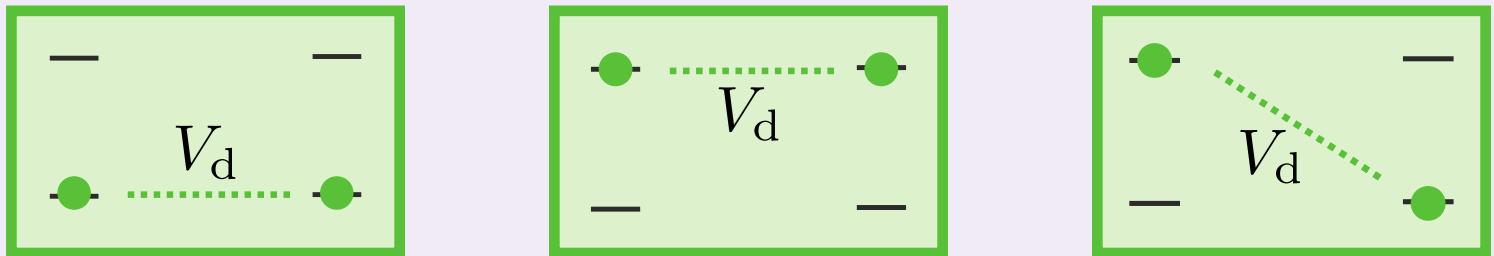
$$\begin{array}{c} |\uparrow\rangle \\ \hline \hline \\ |\downarrow\rangle \end{array} \quad d_{\uparrow\uparrow} = \langle \uparrow | d_z | \uparrow \rangle$$
$$d_{\downarrow\downarrow} = \langle \downarrow | d_z | \downarrow \rangle$$

realizing spin models



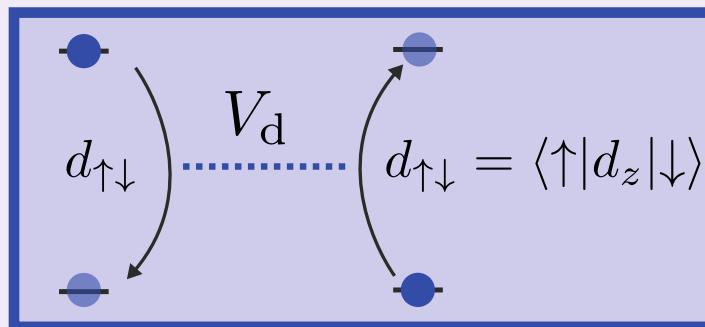
- ▷ 2D optical lattice
- ▷ 1 molecule per site
- ▷ suppressed tunneling

realizing spin-spin interactions

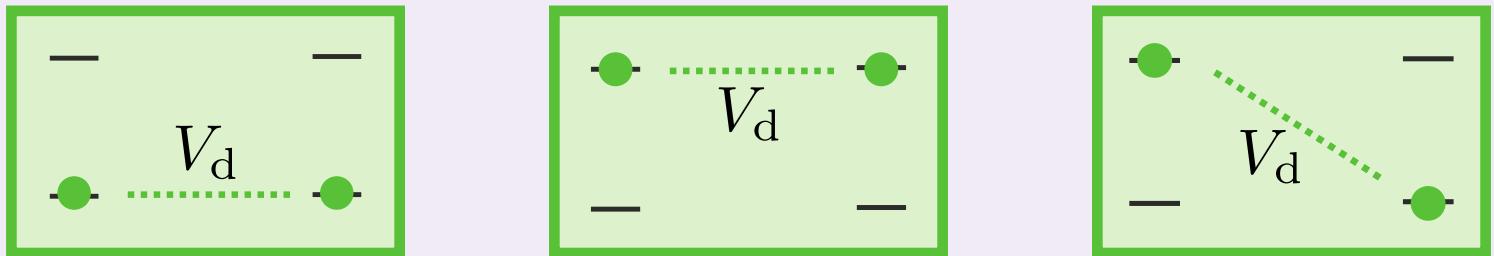


$$\frac{\mathbf{d}_i \mathbf{d}_j}{R_{ij}^3} = \frac{1}{R_{ij}^3} \left[d_{\downarrow\downarrow}^2 \mathcal{P}_i^\downarrow \mathcal{P}_j^\downarrow + d_{\uparrow\uparrow}^2 \mathcal{P}_i^\uparrow \mathcal{P}_j^\uparrow + \left(d_{\uparrow\uparrow} d_{\downarrow\downarrow} \mathcal{P}_i^\uparrow \mathcal{P}_j^\downarrow + \dots \right) \right]$$

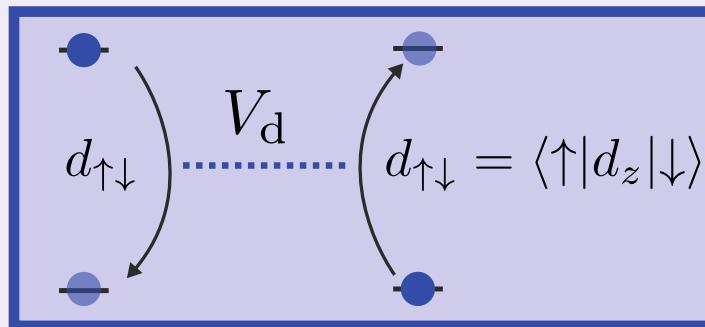
$$d_{\uparrow\downarrow}^2 \left(\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+ \right)$$



realizing spin-spin interactions



$$\frac{\mathbf{d}_i \mathbf{d}_j}{R_{ij}^3} = \frac{1}{R_{ij}^3} \left[J_z \sigma_i^z \sigma_j^z + J_{\perp} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y) \right] + \dots$$

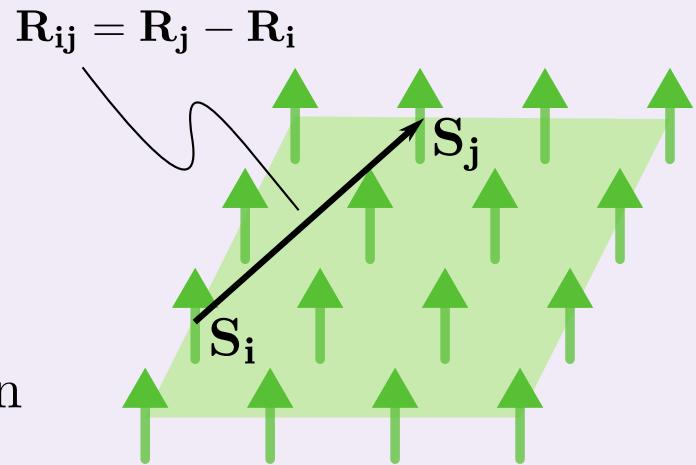


2D spin system

2D lattice spin model

spin couplings: $J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$

$$J_{ij} = J \begin{cases} \delta_{|\mathbf{R}_{ij}|,1} & \text{nearest neighbor} \\ |\mathbf{R}_{ij}|^{-3} & \text{dipolar interaction} \end{cases}$$



add
anisotropy

$$\longrightarrow J_{ij} [\cos(\theta) S_i^z S_j^z + \sin(\theta) (S_i^x S_j^x + S_i^y S_j^y)]$$

NN model

nearest neighbor model:

$$H = J \sum_{\langle i,j \rangle} [\cos \theta S_i^z S_j^z + \sin \theta (S_i^x S_j^x + S_i^y S_j^y)]$$

$$\theta = 0, \pi$$



$$H = \pm J \sum_{\langle i,j \rangle} S_i^z S_j^z$$

(Anti)ferromagnetic Ising model

$$\theta = \pm \pi/2$$

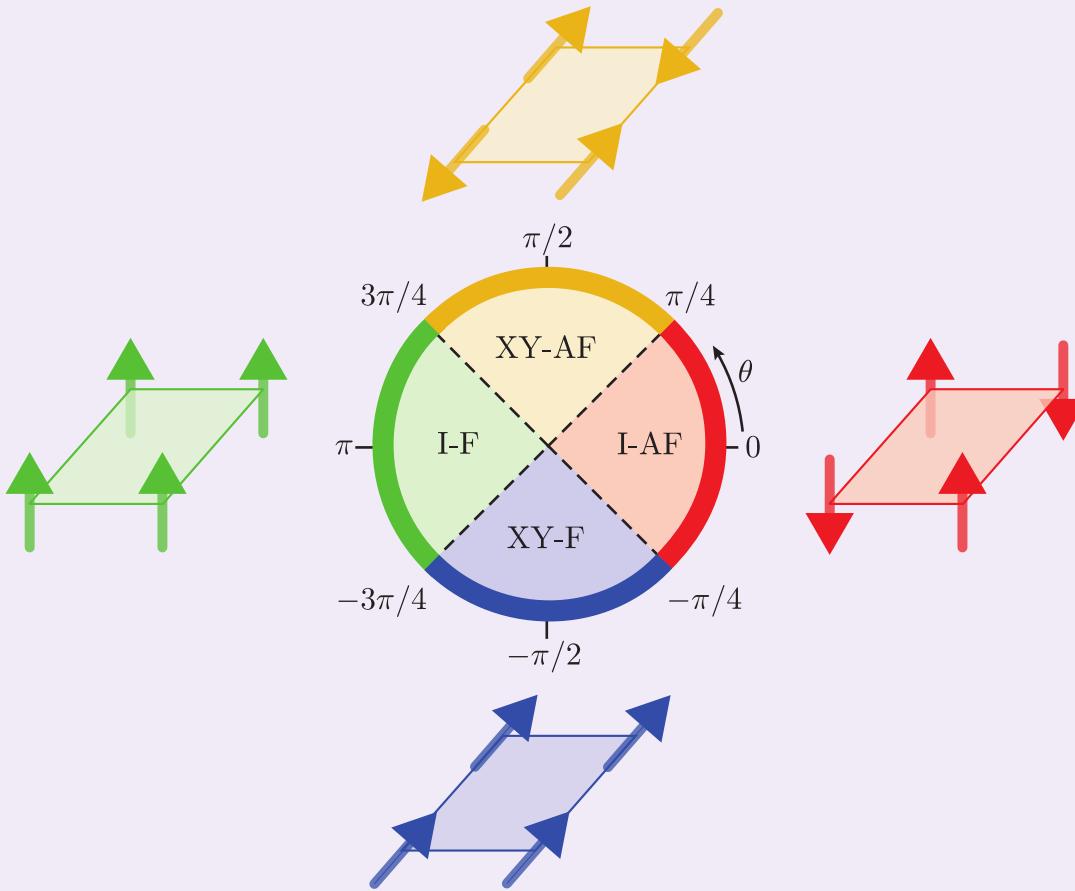


$$H = \pm J \sum_{\langle i,j \rangle} (S_i^x S_j^x + S_i^y S_j^y)$$

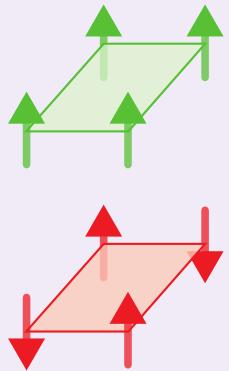
(Anti)ferromagnetic XY model

$\theta = \pi/4$ and $\theta = -3\pi/4$: full $SU(2)$ symmetry (Heisenberg model)

NN phase diagram

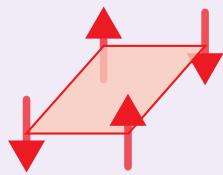
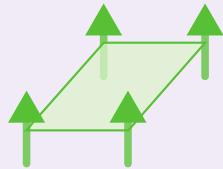


NN phases

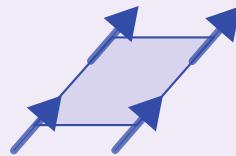


broken \mathbb{Z}_2 symmetry
gapped excitation spectrum
spin wave dispersion $\sim q^2$ for $q \rightarrow 0$

NN phases

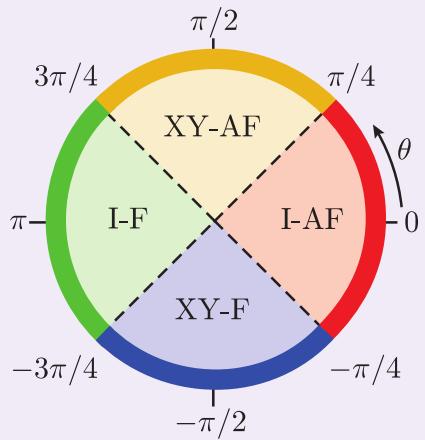


broken \mathbb{Z}_2 symmetry
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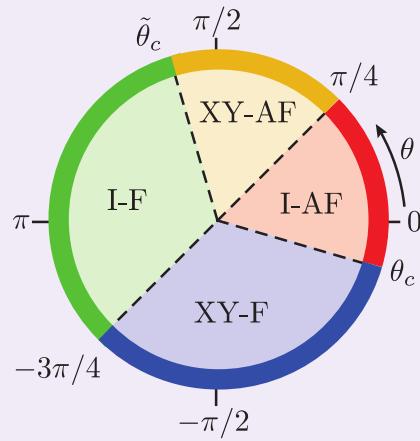


broken $U(1)$ symmetry (only at $T = 0$)
gapless Goldstone mode
linear excitation spectrum for $q \rightarrow 0$

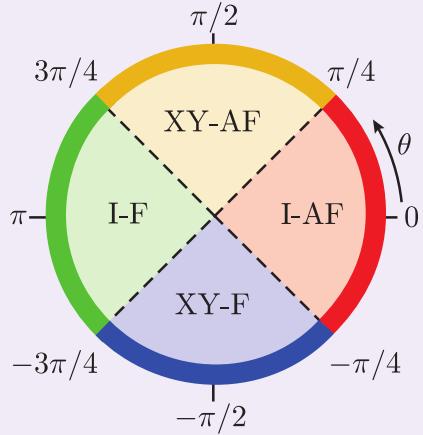
dipolar model: mean-field



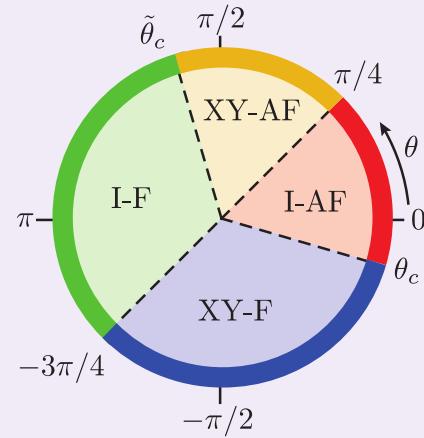
dipolar interactions



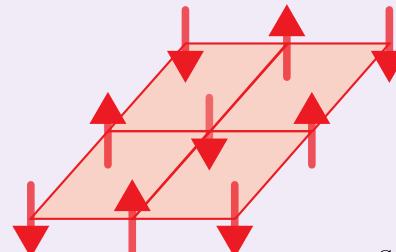
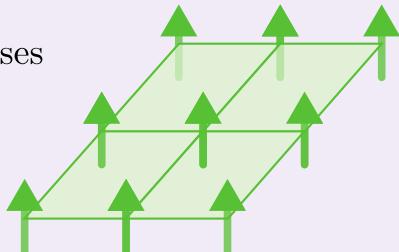
dipolar model: mean-field



dipolar interactions



ferromagnetic phases
are enhanced



second-nearest neighbors
add (weak) frustration

hard-core boson mapping

$$S_i^+ \mapsto a_i^\dagger$$

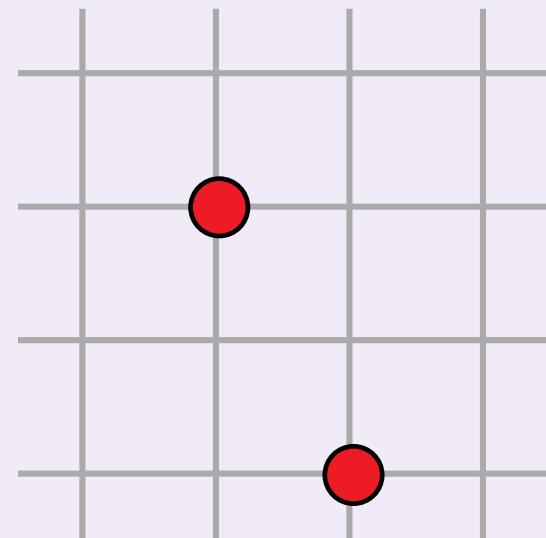
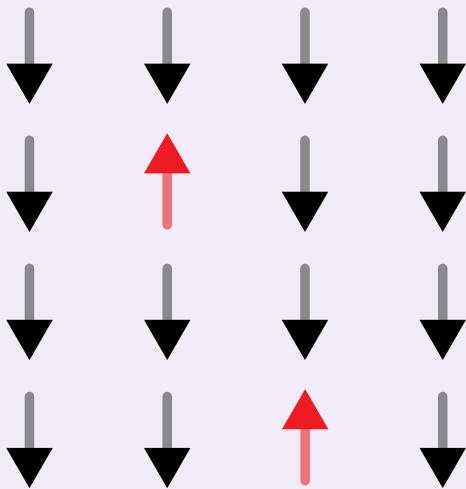
only for $\langle n_i \rangle \ll 1$

$$S_i^- \mapsto a_i$$

$$[a_i, a_j^\dagger] = \delta_{ij}$$

$$S_i^z \mapsto a_i^\dagger a_i - 1/2$$

$$a_i a_i = a_i^\dagger a_i^\dagger = 0$$



spin wave analysis: overview

$$H = Ja^3 \sum_{i \neq j} \frac{\cos \theta S_i^z S_j^z + \frac{1}{2} \sin \theta (S_i^+ S_j^- + S_i^- S_j^+)}{|\mathbf{R}_i - \mathbf{R}_j|^3}$$

Holstein-Primakoff



neglect interaction $n_i n_j \approx 0$

$$H = Ja^3 \sum_{i \neq j} \frac{\cos \theta (n_i n_j - n_i + 1/4) + \frac{1}{2} \sin \theta (a_i^\dagger a_j + a_j^\dagger a_i)}{|\mathbf{R}_i - \mathbf{R}_j|^3}$$

Fourier-Transform



„long-range tunneling“

$$H = \frac{3JN_s\epsilon_0}{4} \cos \theta + \sum_{\mathbf{q}} J(\sin \theta \epsilon_{\mathbf{q}} - \cos \theta \epsilon_0) \left(n_{\mathbf{q}} + \frac{1}{2} \right)$$

other phases: additional Bogoliubov transformation

dipolar dispersion relation

dimensional reasoning

$$\epsilon_{\mathbf{q}} = \sum_{j \neq 0} e^{i\mathbf{R}_j \cdot \mathbf{q}} \frac{a^3}{|\mathbf{R}_j|^3}$$

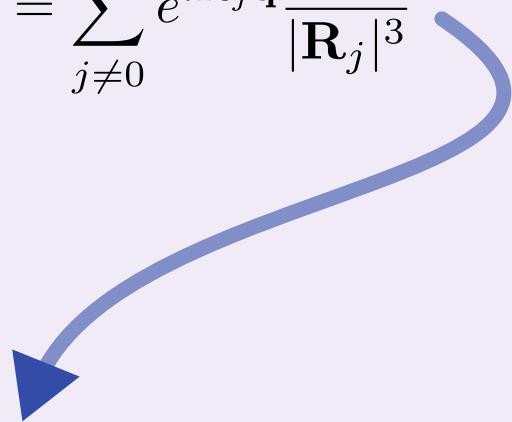
$$\epsilon_{\mathbf{q}} \sim \int d^d r \frac{e^{i\mathbf{q} \cdot \mathbf{r}}}{r^3} \sim q^{3-d}$$

dipolar dispersion relation

dimensional reasoning

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$$\epsilon_{\mathbf{q}} = -2\pi a |\mathbf{q}| \operatorname{erfc}(a|\mathbf{q}|/2\sqrt{\pi}) + 4\pi \left(e^{-\frac{a^2 |\mathbf{q}|^2}{4\pi}} - \frac{1}{3} \right)$$

$$+ 2\pi \sum_{i \neq 0} \int_1^\infty \frac{d\lambda}{\lambda^{3/2}} \left[e^{-\pi\lambda \left(\frac{\mathbf{R}_i}{a} + \frac{a\mathbf{q}}{2\pi} \right)^2} + \lambda^2 e^{-\frac{\pi\lambda |\mathbf{R}_i|^2}{a^2} + i\mathbf{R}_i \cdot \mathbf{q}} \right]$$

dipolar dispersion relation

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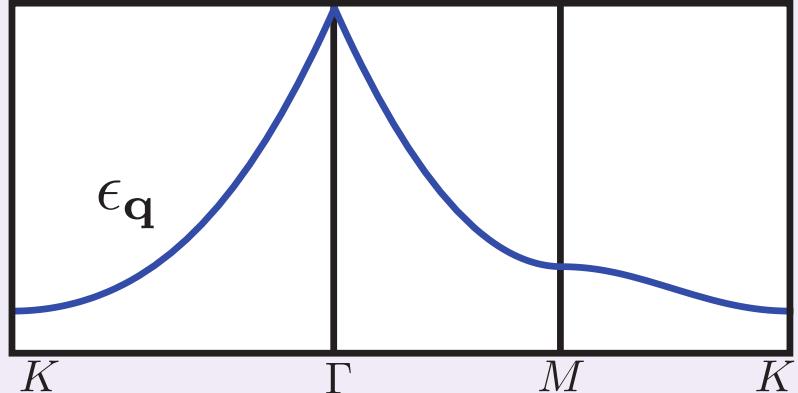
dipolar dispersion relation

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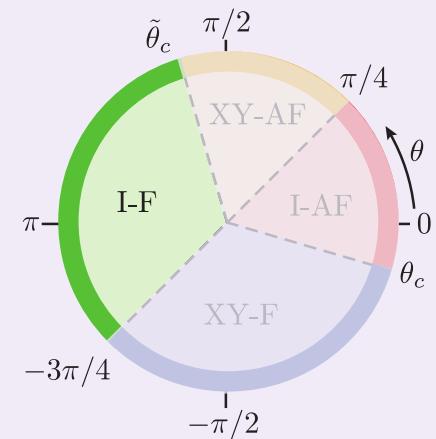
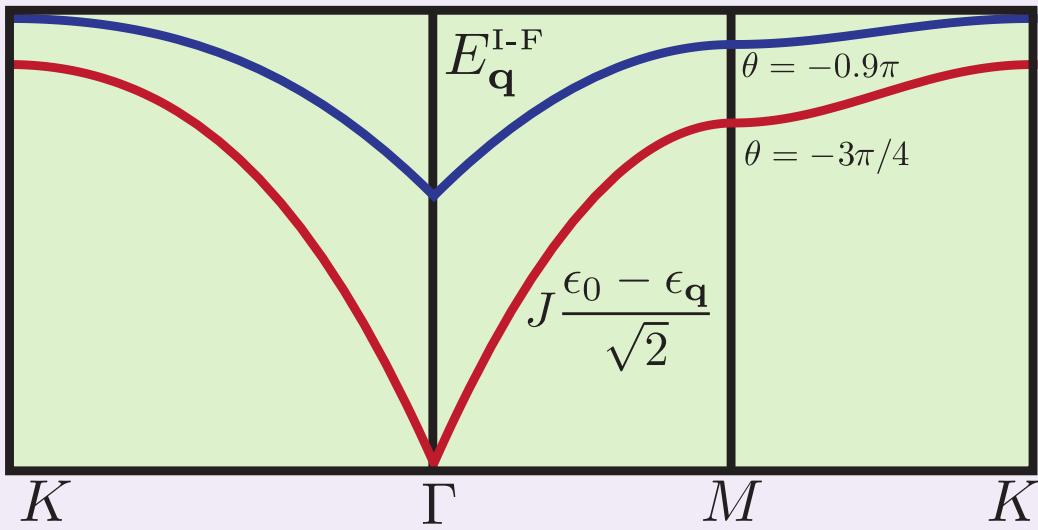
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the I-F phase: dispersion

true long range order at low T

gapped excitation spectrum with
 $E_{\mathbf{q}}^{\text{I-F}} = E_{\Gamma}^{\text{I-F}} + \hbar c |\mathbf{q}|$ for $q \rightarrow 0$

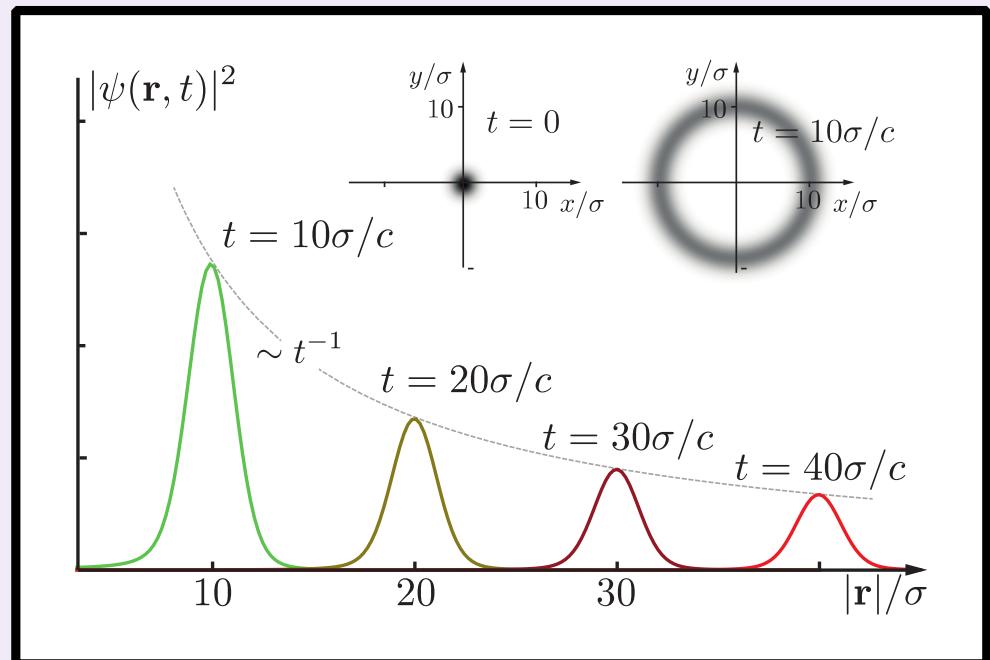


$$\begin{aligned}\Gamma &= (0, 0) \\ M &= (0, \pi/a) \\ K &= (\pi/a, \pi/a)\end{aligned}$$

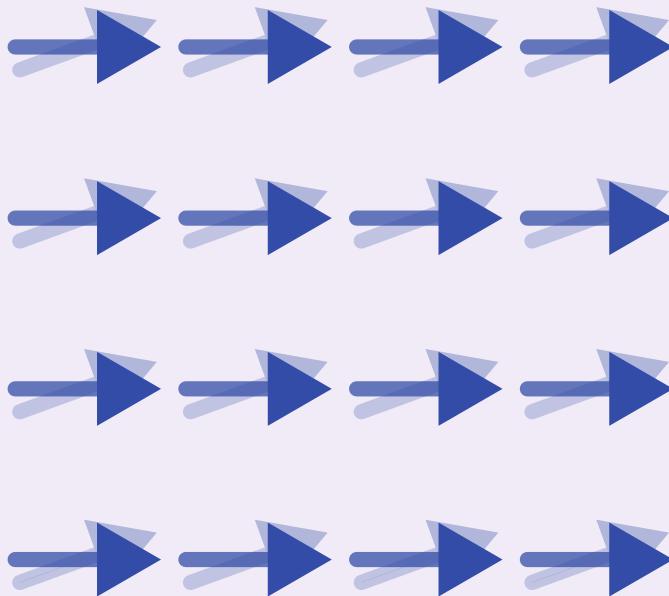
the I-F phase: dynamics

linear dispersion $E_{\mathbf{q}}^{\text{I-F}} \sim E_0^{\text{I-F}} + \hbar c |\mathbf{q}|$

$$c = -2\pi a J \sin \theta$$



the XY-F phase: order?



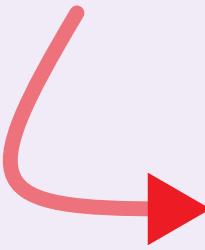
continuous $U(1)$ symmetry

Mermin-Wagner theorem

A continuous symmetry cannot be spontaneously broken in $d \leq 2$ dimensions at finite temperature
(for *sufficiently* short-range interactions)

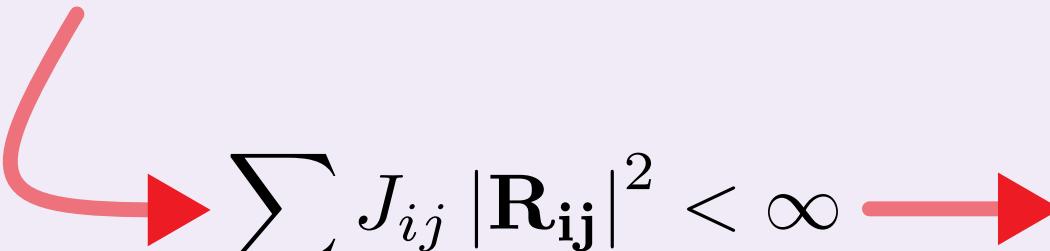
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A continuous symmetry cannot be spontaneously broken in $d \leq 2$ dimensions at finite temperature
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$$\sum_{j \neq i} J_{ij} |\mathbf{R}_{ij}|^2 < \infty$$

Mermin-Wagner theorem

A continuous symmetry cannot be spontaneously broken in $d \leq 2$ dimensions at finite temperature
(for *sufficiently* short-range interactions)


$$\sum_{j \neq i} J_{ij} |\mathbf{R}_{\mathbf{ij}}|^2 < \infty \rightarrow J_{ij} \sim |\mathbf{R}_{\mathbf{ij}}|^{-\alpha}$$
$$\alpha > d + 2$$

the XY-F phase: order!

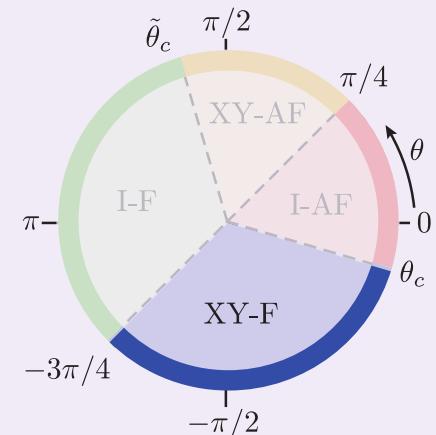
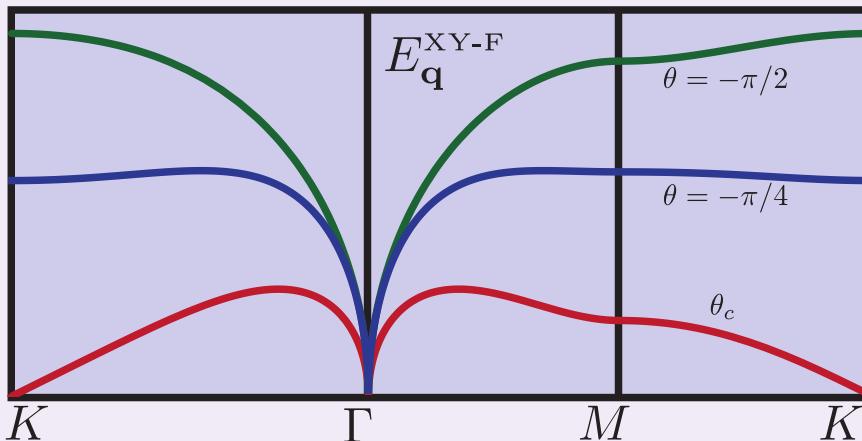
true long-range order at low T:

$$\langle S_i^x S_j^x + S_i^y S_j^y \rangle \rightarrow m^2$$

▷ spontaneously broken $U(1)$ symmetry

dispersion relation for $q \rightarrow 0$

▷ $E_{\mathbf{q}}^{\text{XY-F}} \sim \sqrt{q}$

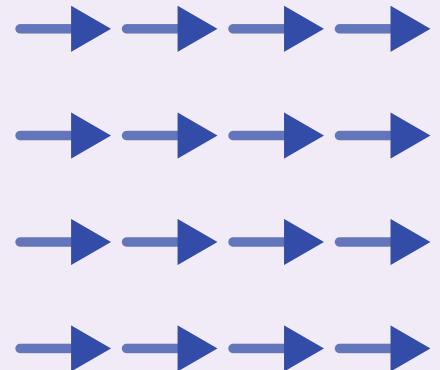


the XY-F phase: justification

suppression of the order parameter

$$\Delta = \langle S_i^x \rangle - 1/2 = \langle a_i^\dagger a_i \rangle \quad \rightarrow \infty \text{ for NN model}$$

$$\Delta \stackrel{\theta = -\frac{\pi}{2}}{=} \begin{cases} 0.08 & T = 0 \\ \text{finite} & T > 0 \end{cases}$$



dipolar interaction favors
mean-field solution due to
additional „neighbors“

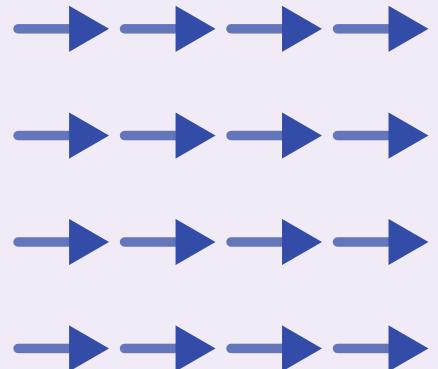
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long-range ferromagnetic
order at finite T



dipolar interaction favors
mean-field solution due to
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the XY-F phase: dynamics

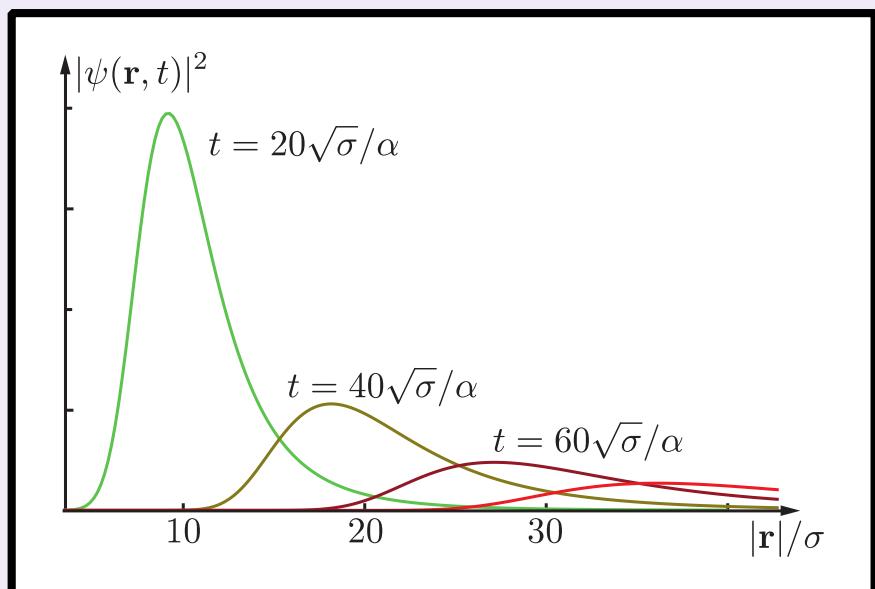
dispersion $E_{\mathbf{q}}^{\text{XY-F}} \sim \sqrt{q}$



group velocity $v_{\mathbf{q}} \sim 1/\sqrt{q}$

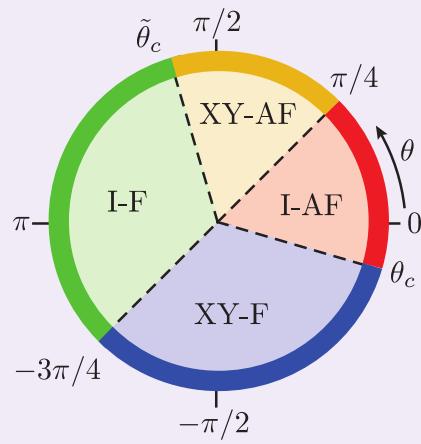


speed of wave
packet $\sim \sqrt{\sigma}$



summary

- gapped linear excitation spectrum
- algebraic correlations

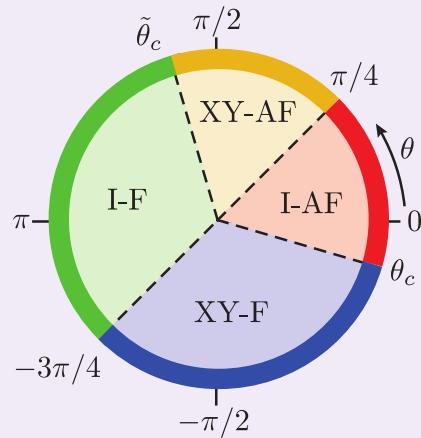


- excitation spectrum $\sim \sqrt{q}$
- spontaneously broken $U(1)$ symmetry in 2D
- ferromagnetically ordered state at finite T

summary

- conventional linear Goldstone mode
- Kosterlitz-Thouless transition to quasi-ordered state

- gapped linear excitation spectrum
- algebraic correlations



- gapped linear excitation spectrum
- algebraic correlations

- excitation spectrum $\sim \sqrt{q}$
- spontaneously broken $U(1)$ symmetry in 2D
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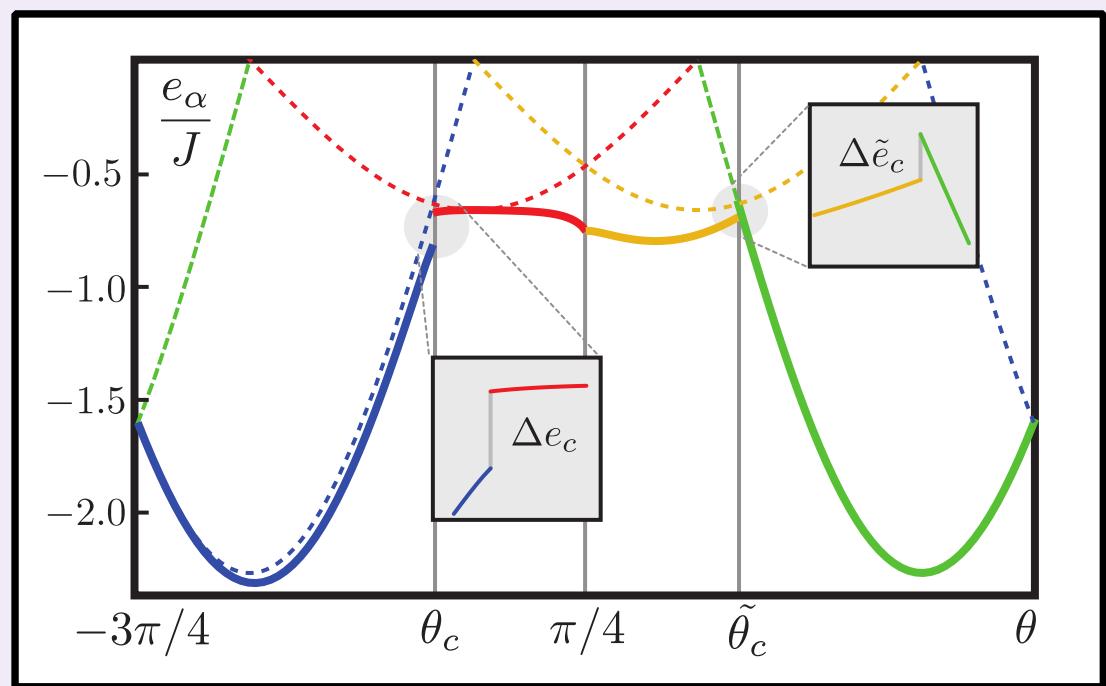
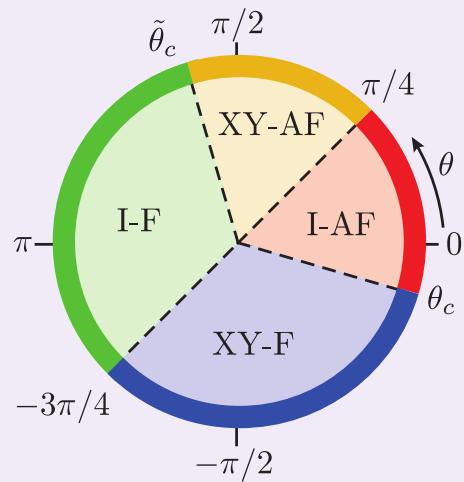
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spin wave analysis: modifications



the XY-F phase: correlations

correlation function	$T = 0$	$0 < T < T_c$	$T_c < T$
$\langle S_i^z S_j^z \rangle$	$\sim \mathbf{r} ^{-5/2}$	$\sim \mathbf{r} ^{-3}$	$\sim \mathbf{r} ^{-3}$
$\langle S_i^y S_j^y + S_i^x S_j^x \rangle - m^2$	$\sim \mathbf{r} ^{-3/2}$	$\sim \mathbf{r} ^{-1}$	$\sim \mathbf{r} ^{-3}$

