David Peter

Ulm, July 11th, 2012

spin systems with dipolar interactions

motivation

polar molecules
in 2D optical lattices

2D dipolar
spin systems

truly new behavior due
to dipolar interactions?

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ARTICLES

A toolbox for lattice-spin models with polar molecules

A. MICHELI*, G. K. BRENNEN AND P. ZOLLER
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2D dipolar
spin systems

understanding of solid-state systems often based on short-range models

truly new behavior due to dipolar interactions?

outline

introduction
- 2D lattice spin models
- reminder: nearest neighbor interactions

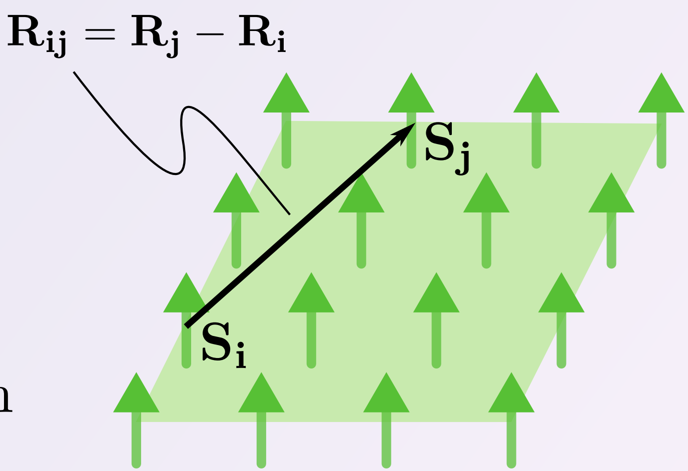
dipolar spin model
- mean field prediction
- spin wave analysis

ferromagnetic Ising and XY phases
- excitations spectra
- long-range order
- spin wave dynamics

2D spin system

2D lattice spin model
spin couplings: $J_{ij}\mathbf{S_i}\mathbf{S_j}$

$$J_{ij} = J \begin{cases} \delta_{|\mathbf{R_{ij}}|,1} & \text{nearest neighbor} \\ |\mathbf{R_{ij}}|^{-3} & \text{dipolar interaction} \end{cases}$$



add
anisotropy

$$\longrightarrow \quad J_{ij}\left[\cos(\theta)\, S_i^z S_j^z + \sin(\theta)\left(S_i^x S_j^x + S_i^y S_j^y\right)\right]$$

NN model

nearest neighbor model:

$$H = J \sum_{\langle i,j \rangle} \left[\cos\theta \, S_i^z S_j^z + \sin\theta \left(S_i^x S_j^x + S_i^y S_j^y \right) \right]$$

$\theta = 0, \pi$

$\theta = \pm\pi/2$

$$H = \pm J \sum_{\langle i,j \rangle} S_i^z S_j^z$$

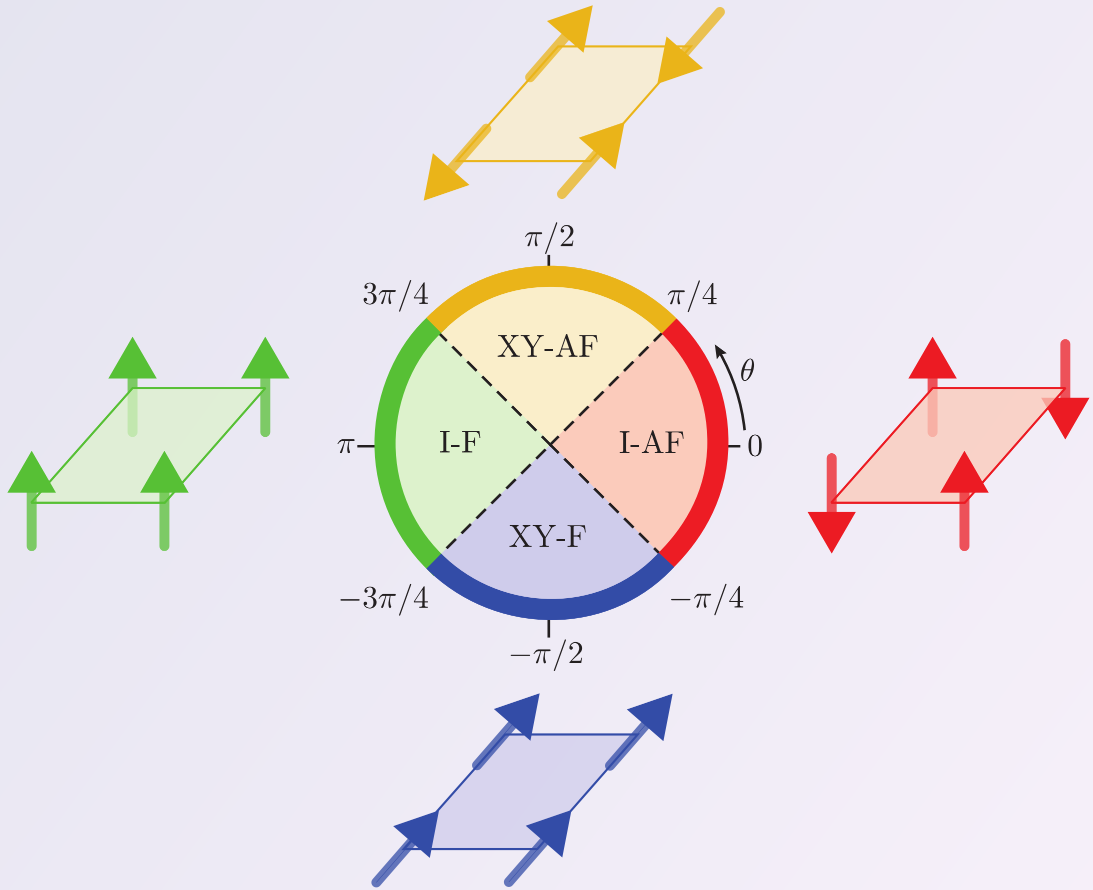
(Anti)ferromagnetic Ising model

$$H = \pm J \sum_{\langle i,j \rangle} \left(S_i^x S_j^x + S_i^y S_j^y \right)$$

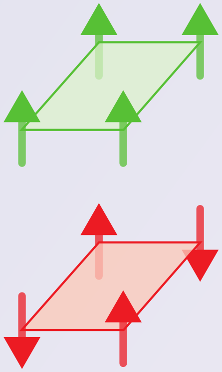
(Anti)ferromagnetic XY model

$\theta = \pi/4$ and $\theta = -3\pi/4$: full $SU(2)$ symmetry (Heisenberg model)

NN phase diagram

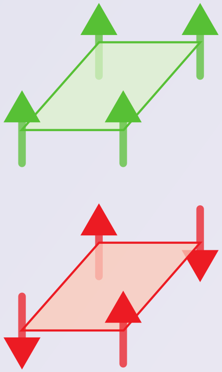


NN phases



broken $\mathbb{Z}_2$ symmetry
gapped excitation spectrum
spin wave dispersion $\sim q^2$ for $q \to 0$

NN phases



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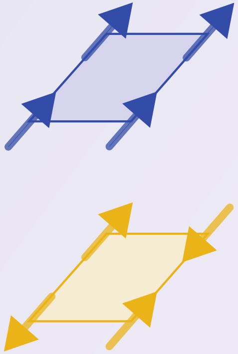
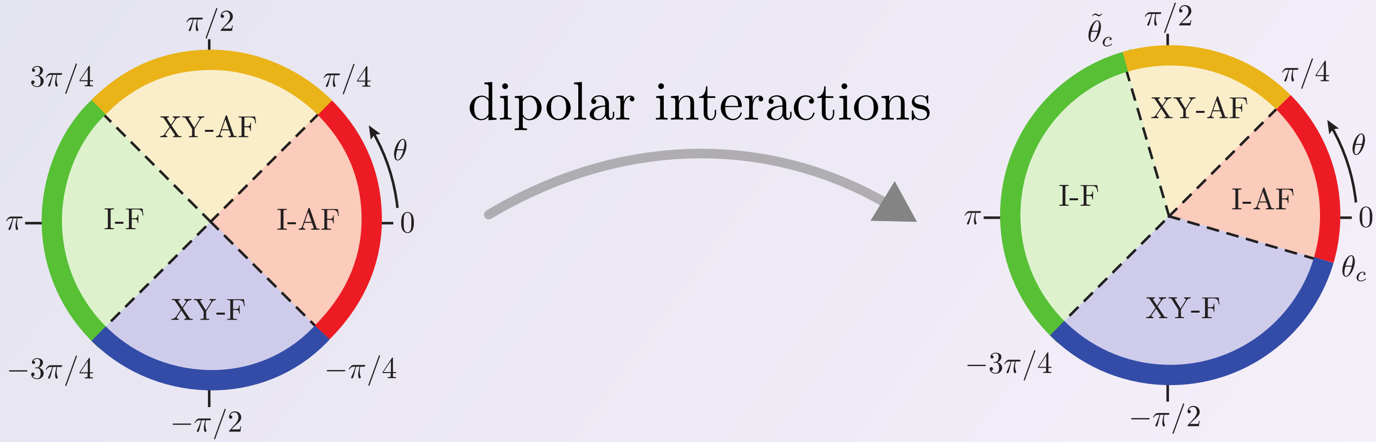
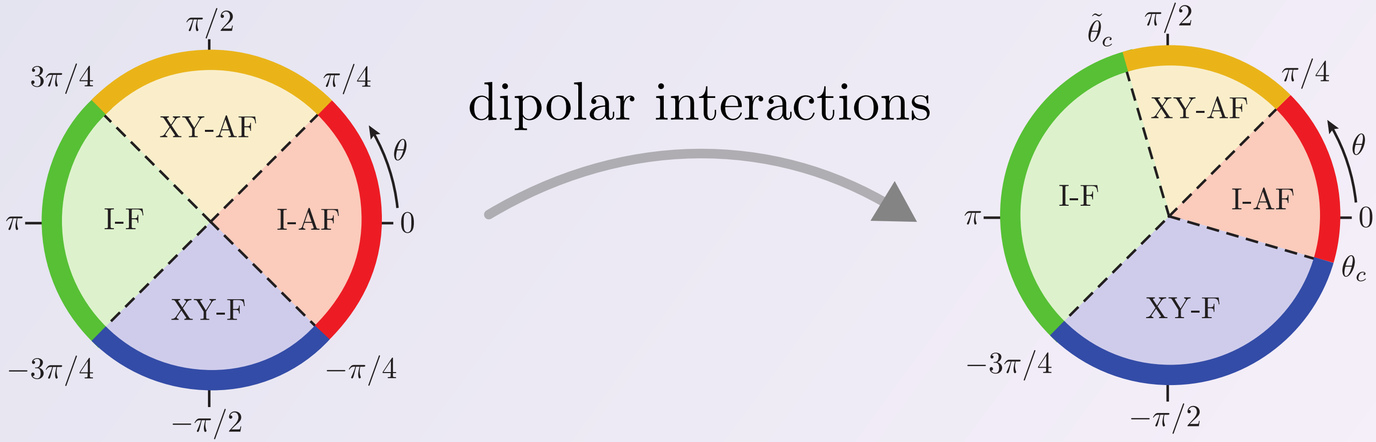


broken $U(1)$ symmetry (only at $T = 0$)
gapless Goldstone mode
linear excitation spectrum for $q \to 0$

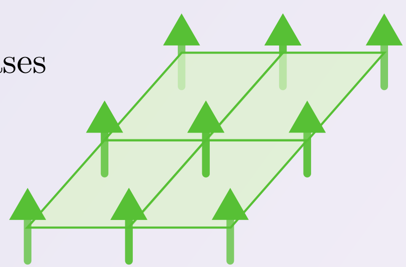
dipolar model: mean-field

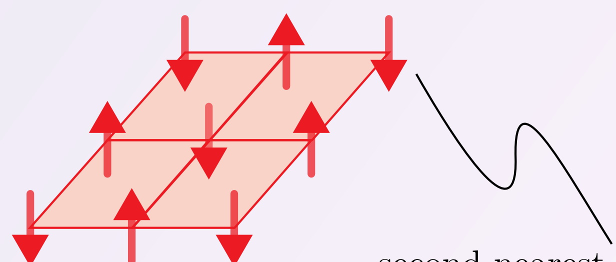


dipolar model: mean-field



ferromagnetic phases
are enhanced





second-nearest neighbors
add (weak) frustration

dipolar dispersion relation

dimensional reasoning

$$\epsilon_{\mathbf{q}} = \sum_{j \neq 0} e^{i\mathbf{R}_j \mathbf{q}} \frac{a^3}{|\mathbf{R}_j|^3}$$

$$\epsilon_{\mathbf{q}} \sim \int \mathrm{d}^d r \, \frac{e^{i\mathbf{q}\mathbf{r}}}{r^3} \sim q^{3-d}$$

dipolar dispersion relation

dimensional reasoning

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simplify

dipolar dispersion relation

dimensional reasoning

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simplify

$$\epsilon_{\mathbf{q}} = -\,2\pi a|\mathbf{q}|\,\mathrm{erfc}(a|\mathbf{q}|/2\sqrt{\pi}) + 4\pi\left(e^{-\frac{a^2|\mathbf{q}|^2}{4\pi}} - \frac{1}{3}\right)$$
$$+\,2\pi\sum_{i\neq 0}\int_1^\infty \frac{d\lambda}{\lambda^{3/2}}\left[e^{-\pi\lambda\left(\frac{\mathbf{R}_i}{a}+\frac{a\mathbf{q}}{2\pi}\right)^2} + \lambda^2 e^{-\frac{\pi\lambda|\mathbf{R}_i|^2}{a^2}+i\mathbf{R}_i\mathbf{q}}\right]$$

dipolar dispersion relation

dimensional reasoning

$$\epsilon_{\mathbf{q}} \sim \int \mathrm{d}^d r \, \frac{e^{i\mathbf{q}\mathbf{r}}}{r^3} \sim q^{3-d}$$

$$\epsilon_{\mathbf{q}} = \sum_{j\neq 0} e^{i\mathbf{R}_j\mathbf{q}} \frac{a^3}{|\mathbf{R}_j|^3}$$

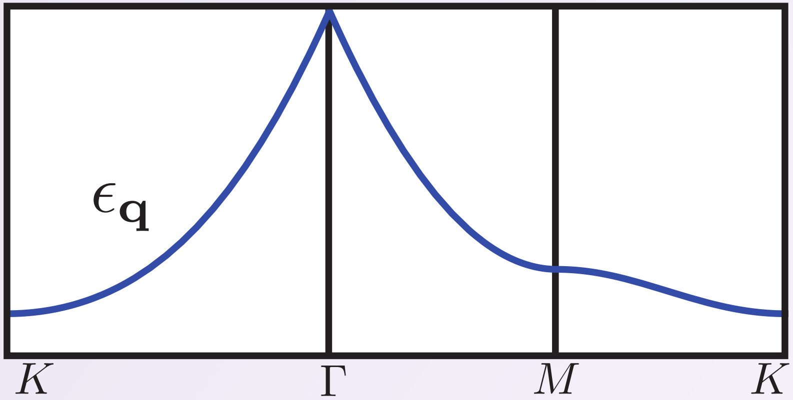
simplify

$$\epsilon_{\mathbf{q}} = \boxed{-2\pi a|\mathbf{q}|} \operatorname{erfc}(a|\mathbf{q}|/2\sqrt{\pi}) + 4\pi\left(e^{-\frac{a^2|\mathbf{q}|^2}{4\pi}} - \frac{1}{3}\right)$$

$$+ 2\pi \sum_{i\neq 0} \int_1^\infty \frac{d\lambda}{\lambda^{3/2}} \left[e^{-\pi\lambda\left(\frac{\mathbf{R}_i}{a} + \frac{a\mathbf{q}}{2\pi}\right)^2} + \lambda^2 e^{-\frac{\pi\lambda|\mathbf{R}_i|^2}{a^2} + i\mathbf{R}_i\mathbf{q}}\right]$$

dipolar dispersion relation

$$\epsilon_{\mathbf{q}} = \sum_{j\neq 0} e^{i\mathbf{R}_j\mathbf{q}} \frac{a^3}{|\mathbf{R}_j|^3}$$



simplify

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hard-core boson mapping

only for $n_i \ll 1$

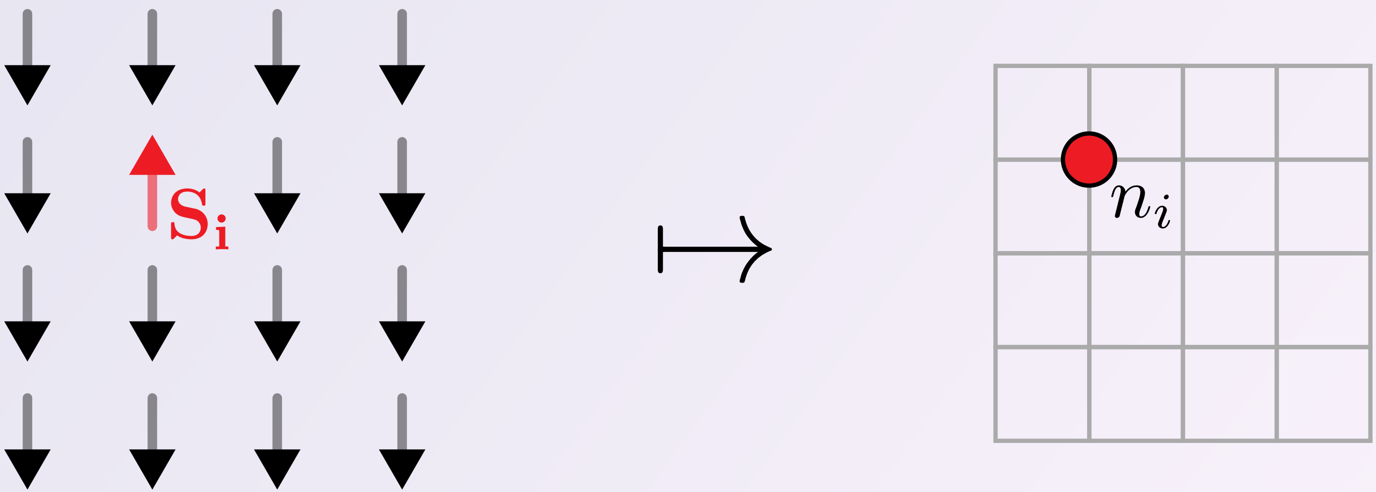
$$S_i^+ \mapsto a_i^\dagger$$

$$S_i^- \mapsto a_i$$

$$S_i^z \mapsto a_i^\dagger a_i - 1/2$$

$$[a_i \,, a_j^\dagger] = \delta_{ij}$$

$$a_i a_i = a_i^\dagger a_i^\dagger = 0$$



spin wave analysis: overview

$$H = Ja^3 \sum_{i \neq j} \frac{\cos\theta \, S_i^z S_j^z + \frac{1}{2}\sin\theta \left(S_i^+ S_j^- + S_i^- S_j^+\right)}{|\mathbf{R}_i - \mathbf{R}_j|^3}$$

Holstein-Primakoff

neglect interaction $n_i n_j \approx 0$

$$H = Ja^3 \sum_{i \neq j} \frac{\cos\theta \, (n_i n_j - n_i + 1/4) + \frac{1}{2}\sin\theta \left(a_i^\dagger a_j + a_j^\dagger a_i\right)}{|\mathbf{R}_i - \mathbf{R}_j|^3}$$

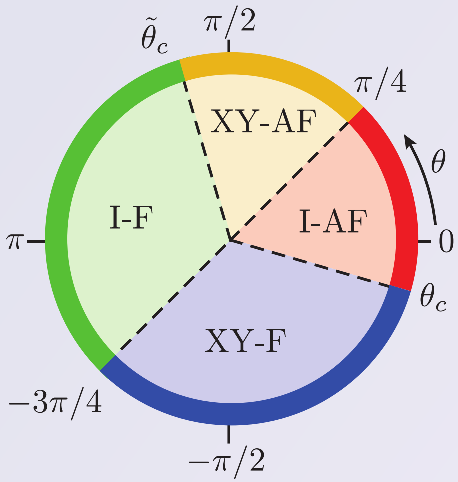
„long-range tunneling“

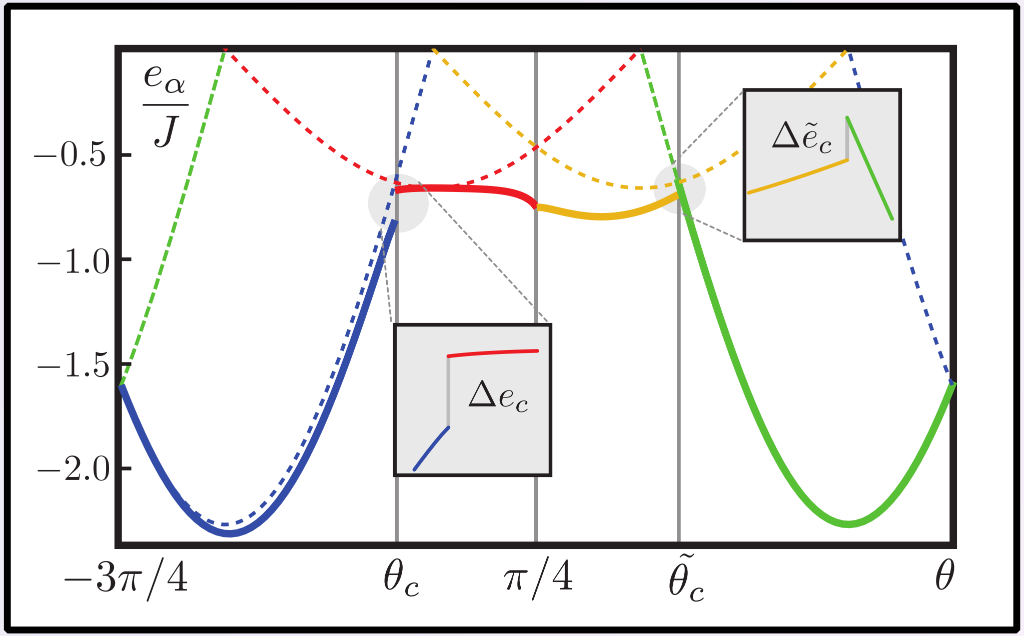
Fourier-Transform

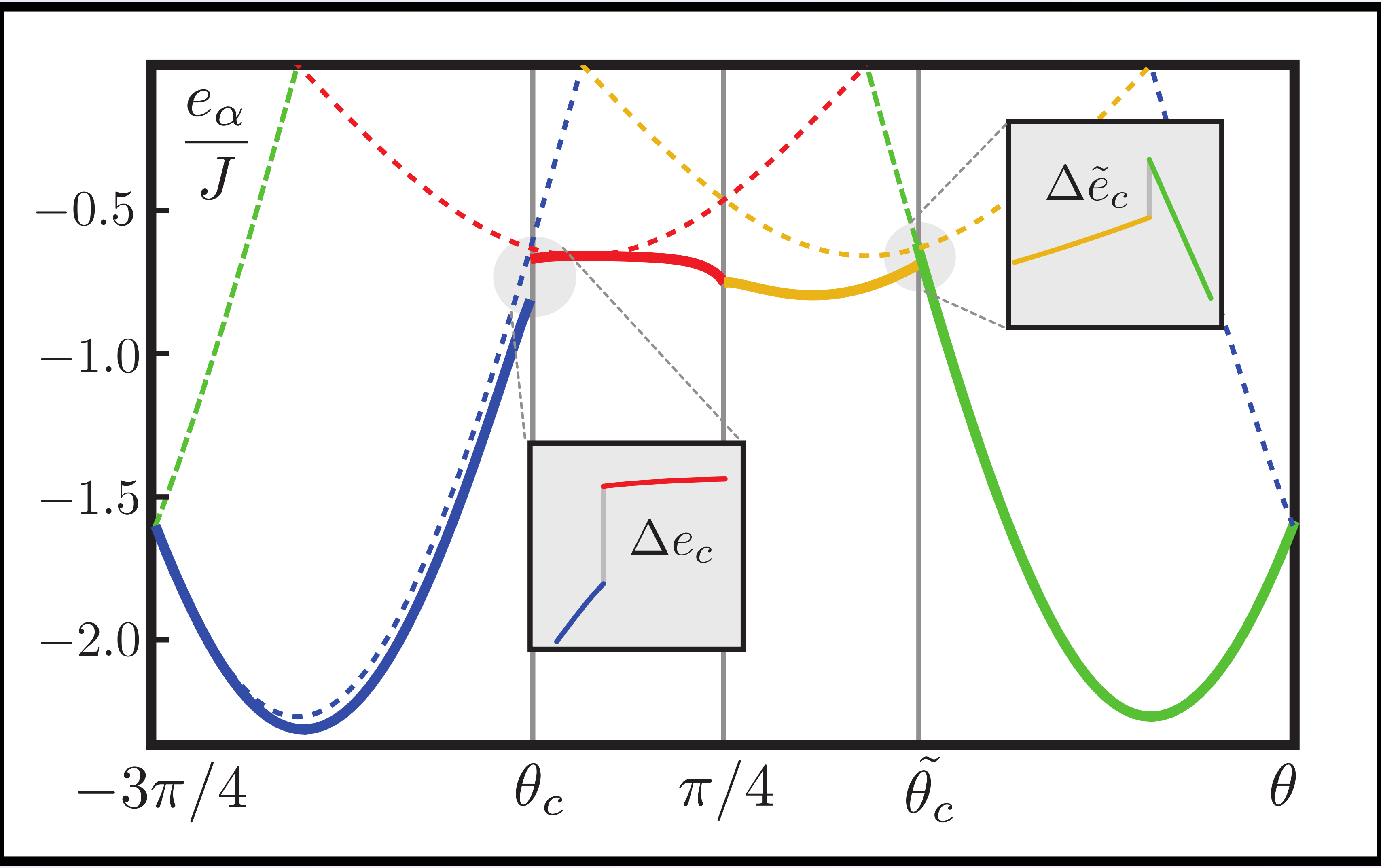
$$H = \frac{3JN_s\epsilon_0}{4}\cos\theta + \sum_{\mathbf{q}} J(\sin\theta \, \epsilon_{\mathbf{q}} - \cos\theta \, \epsilon_0)\left(n_{\mathbf{q}} + \frac{1}{2}\right)$$

other phases: additional Bogoliubov transformation

spin wave analysis: modifications

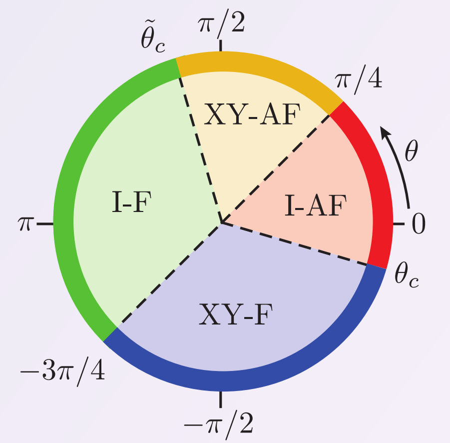






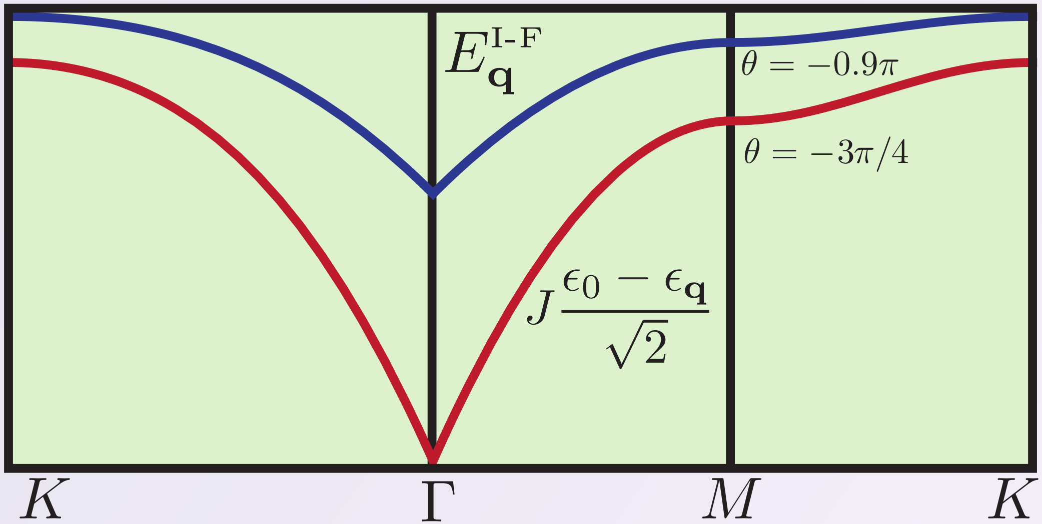
$\frac{e_\alpha}{J}$
-0.5
-1.0
-1.5
-2.0
$-3\pi/4$
θ_c
$\pi/4$
$\tilde{\theta}_c$
θ
Δe_c
$\Delta \tilde{e}_c$

the I-F phase: dispersion



dispersion relation for $q \to 0$

$E_{\mathbf{q}}^{\text{I-F}} \sim q$



$\Gamma = (0, 0)$

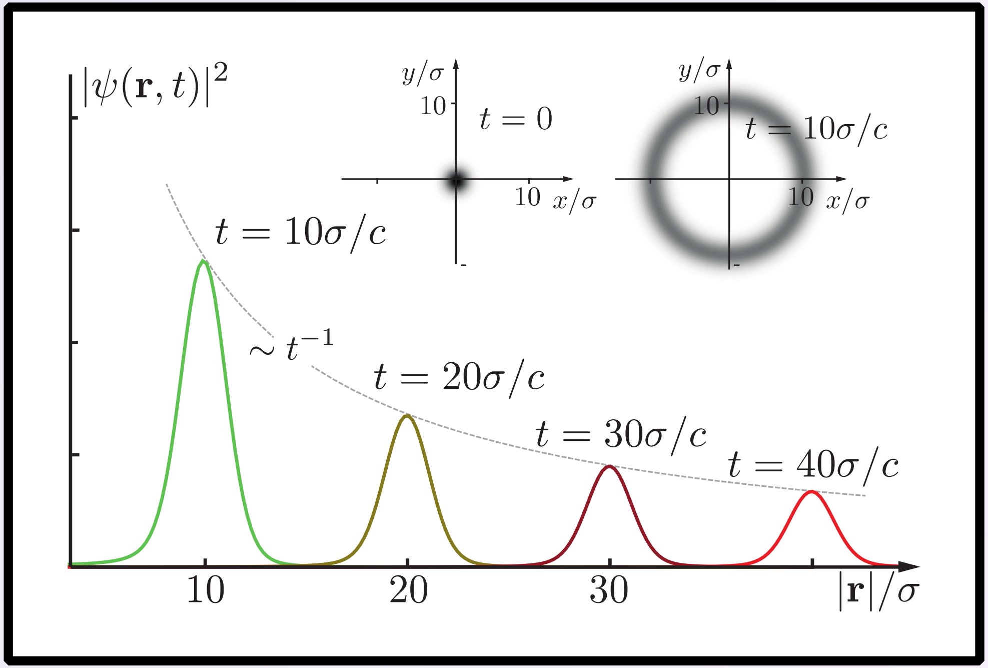
$M = (0, \pi/a)$

$K = (\pi/a, \pi/a)$

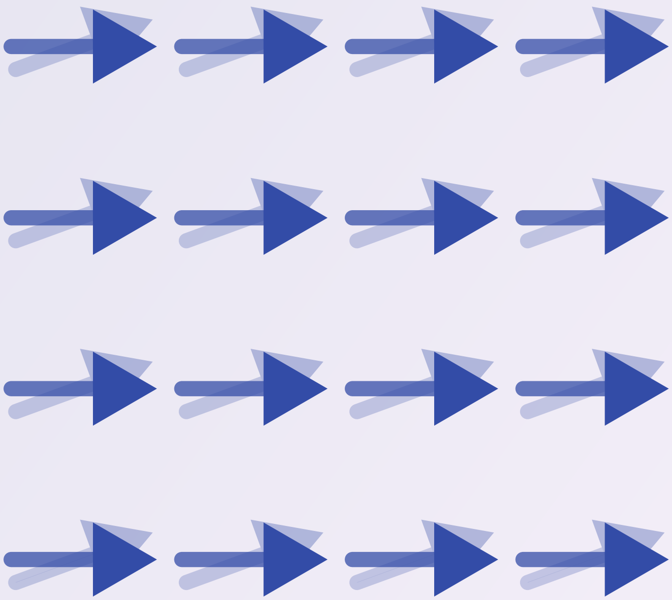
the I-F phase: dynamics

linear dispersion $E_{\mathbf{q}}^{\text{I-F}} \sim E_0^{\text{I-F}} + \hbar c\,|\mathbf{q}|$

$$c = -2\pi a J \sin\theta$$



the XY-F phase: order?



continuous $U(1)$ symmetry

Mermin-Wagner theorem

A continuous symmetry cannot be spontaneously broken in $d \leq 2$ dimensions at finite temperature

(for *sufficiently* short-range interactions)

N.D. Mermin and H. Wagner, Phys. Rev. Lett. **17**, 1133 (1966)

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$$\sum_{j \neq i} J_{ij} \left|\mathbf{R_{ij}}\right|^2 < \infty$$

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A continuous symmetry cannot be spontaneously broken in $d \leq 2$ dimensions at finite temperature

(for *sufficiently* short-range interactions)

$$\sum_{j \neq i} J_{ij} \left|\mathbf{R_{ij}}\right|^2 < \infty \quad \longrightarrow \quad J_{ij} \sim \left|\mathbf{R_{ij}}\right|^{-\alpha}$$

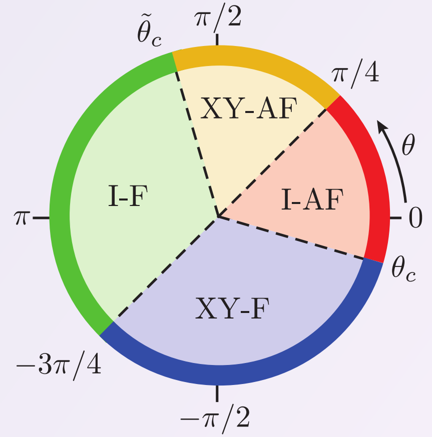
$$\alpha > d + 2$$

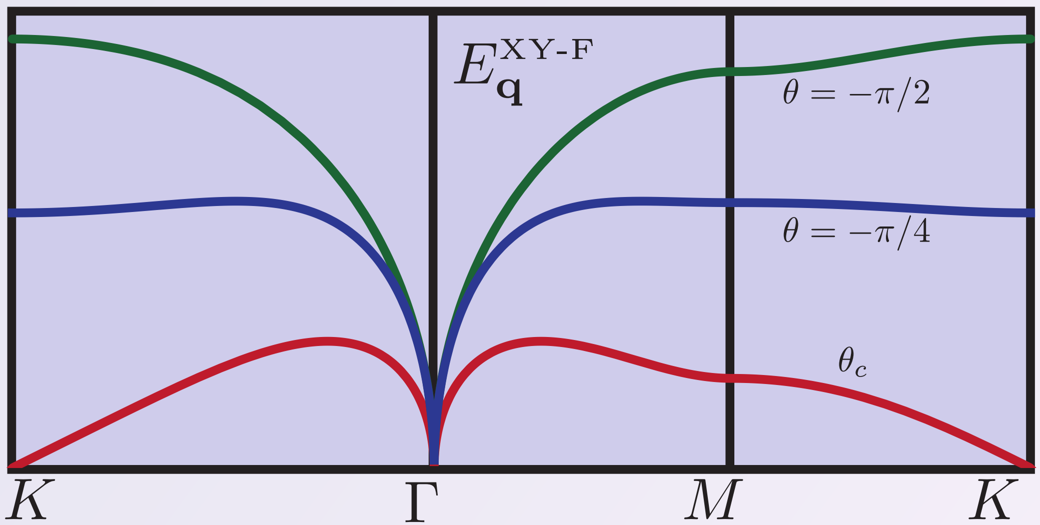
N.D. Mermin and H. Wagner, Phys. Rev. Lett. **17**, 1133 (1966)

the XY-F phase: dispersion

dispersion relation for $q \to 0$

$E_{\mathbf{q}}^{\text{XY-F}} \sim \sqrt{q}$



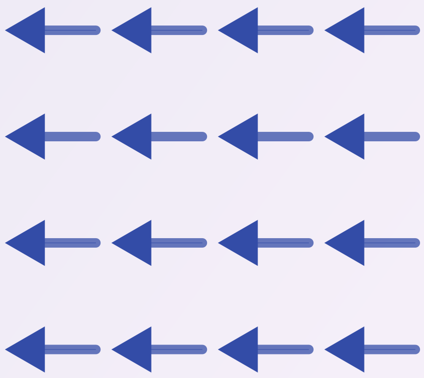


the XY-F phase: order!

suppression of the order parameter

$$\Delta = \langle S_i^x \rangle + 1/2 = \langle a_i^\dagger a_i \rangle \qquad \to \infty \text{ for NN model}$$

$$\Delta \overset{\theta=-\frac{\pi}{2}}{=} \begin{cases} 0.08 & T = 0 \\ \text{finite} & T > 0 \end{cases}$$



dipolar interaction favors mean-field solution due to additional „neighbors“

the XY-F phase: order!

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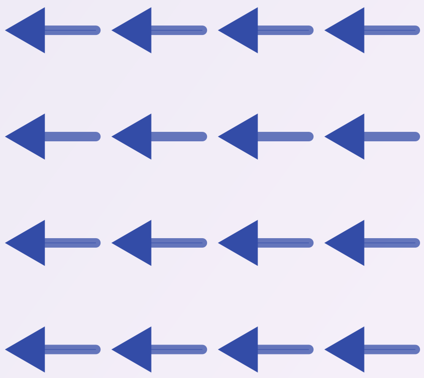


long-range ferromagnetic order at finite T

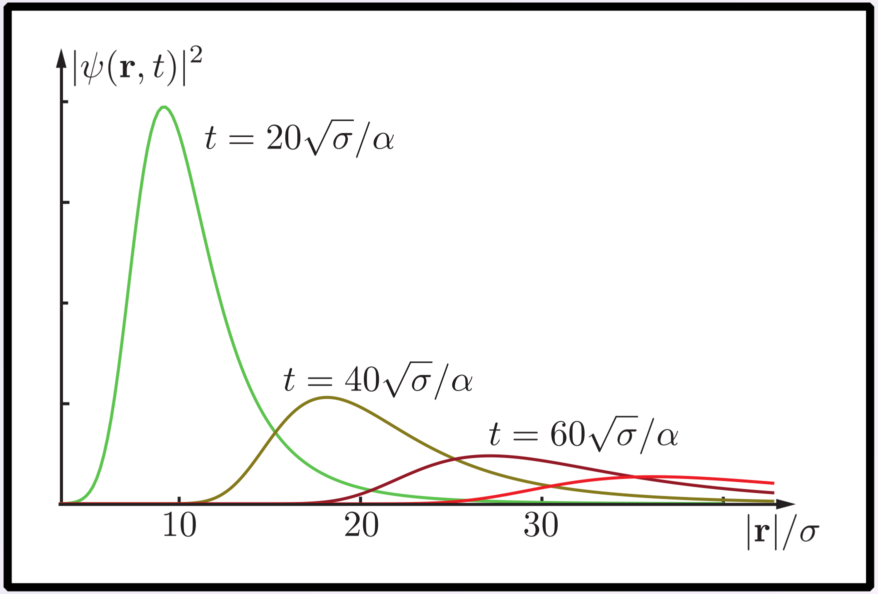
dipolar interaction favors mean-field solution due to additional „neighbors“

the XY-F phase: dynamics

dispersion $E_{\mathbf{q}}^{\text{XY-F}} \sim \sqrt{q}$

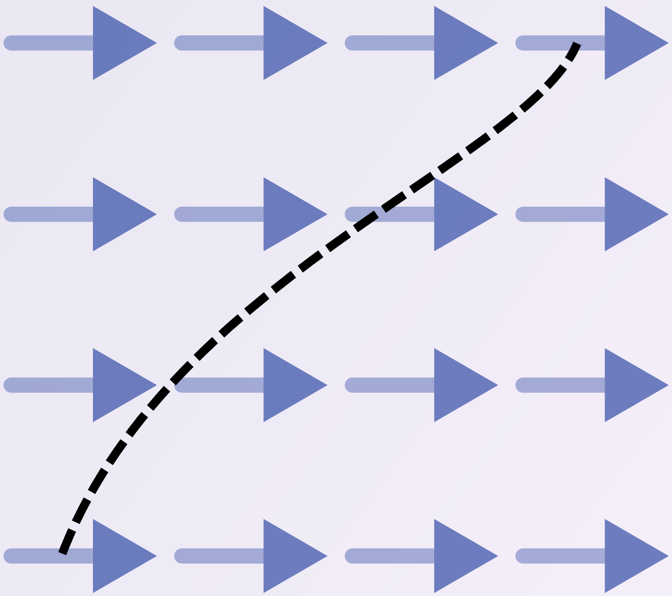
group velocity $v_{\mathbf{q}} \sim 1/\sqrt{q}$

speed of wave packet $\sim \sqrt{\sigma}$



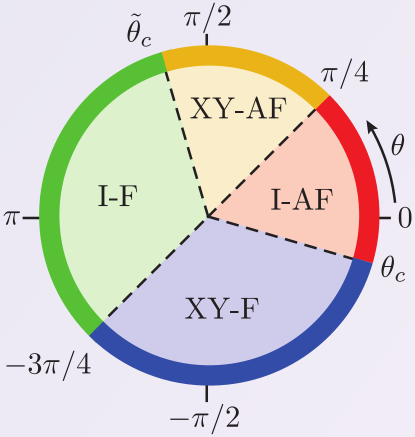
the XY-F phase: correlations

correlation function	$T = 0$	$0 < T < T_c$	$T_c < T$
$\langle S_i^z S_j^z \rangle$	$\sim \lvert\mathbf{r}\rvert^{-5/2}$	$\sim \lvert\mathbf{r}\rvert^{-3}$	$\sim \lvert\mathbf{r}\rvert^{-3}$
$\langle S_i^y S_j^y + S_i^x S_j^x \rangle - m^2$	$\sim \lvert\mathbf{r}\rvert^{-3/2}$	$\sim \lvert\mathbf{r}\rvert^{-1}$	$\sim \lvert\mathbf{r}\rvert^{-3}$



summary

- gapped linear excitation spectrum
- algebraic correlations

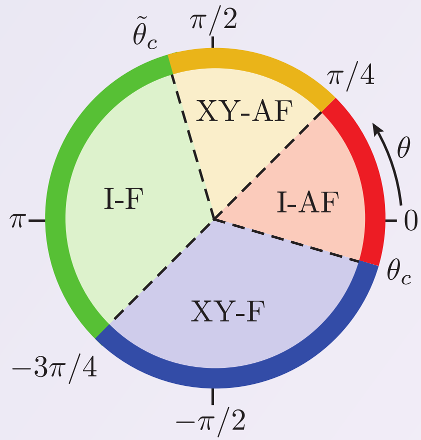


- excitation spectrum $\sim \sqrt{q}$
- spontaneously broken $U(1)$ symmetry in 2D
- ferromagnetically ordered state at finite T

summary

- conventional linear Goldstone mode
- Kosterlitz-Thouless transition to quasi-ordered state

- gapped linear excitation spectrum
- algebraic correlations



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goo.gl/9eyWt

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