Anomalous Behavior of Spin Systems with Dipolar Interactions

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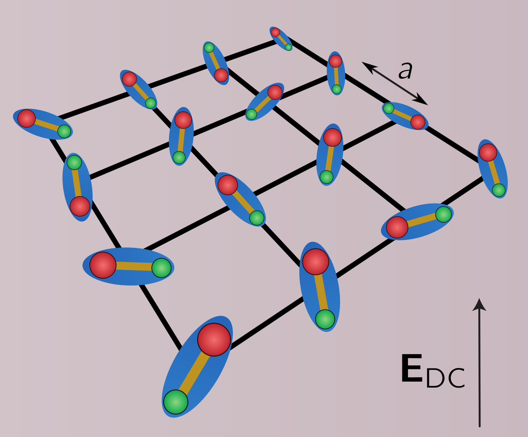
▷ Spin systems realized by cold polar molecules intrinsically interact by dipole-dipole interactions

▷ $1/r^3$ interactions in 2D are exceptional:

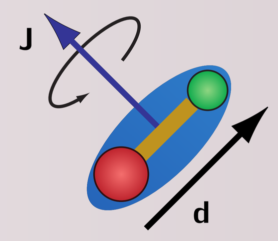
- Fourier-transform $\epsilon_q \sim |\boldsymbol{q}|$ leads to „anomalous" dispersion relations
- Not captured by Mermin-Wagner theorem

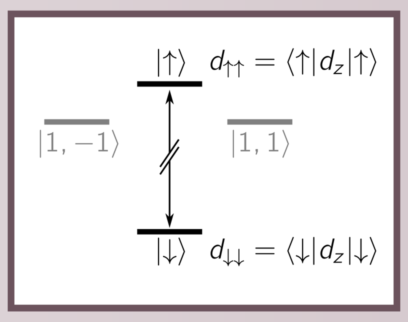


Realization of spin-model (in a nutshell)

$H = BJ^2 - d\boldsymbol{E}$

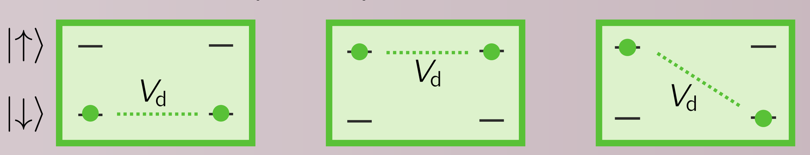
▷ use $|J, M\rangle$ states



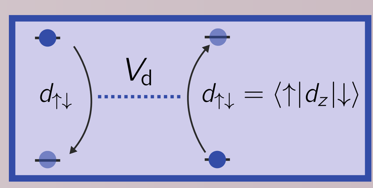


$|\uparrow\rangle = |10\rangle$ and $|\downarrow\rangle = |00\rangle$ states acquire dipole moment due to $\boldsymbol{E}_{\mathrm{DC}}$

dipole-dipole interactions



$$\frac{\mathbf{d}_i \mathbf{d}_j}{R_{ij}^3} = \frac{1}{R_{ij}^3}\left[J_z \sigma_i^z \sigma_j^z + J_\perp(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y)\right] + \ldots$$



Dipolar interactions

▷ Non-analytic behavior of Lattice Fourier transformation for $\boldsymbol{q} \to 0$

$$\epsilon_{\boldsymbol{q}} = \sum_{i \neq 0} \frac{e^{i\boldsymbol{q}\boldsymbol{R}_i}}{R_i^3} = \sum_{i \neq 0}\left[\frac{1}{R_i^3} - \boxed{\frac{(\boldsymbol{q}\boldsymbol{R}_i)^2}{R_i^3}} + \ldots\right] = \epsilon_0 - 2\pi\,|\boldsymbol{q}| + \mathcal{O}(q^2)$$

diverges if order of limits is changed and violates Mermin-Wagner short-range condition

can be obtained by Ewald summation

Mermin-Wagner theorem

A continuous symmetry cannot be spontaneously broken in $d \leq 2$ dimensions at finite temperature … **if** $\boxed{\sum_{j \neq i} J_{ij}\,|R_{ij}|^2}$ converges

Spin-wave analysis

2D Spin-Hamiltonian with dipolar interactions and anisotropy parameter θ

$$H = Ja^3 \sum_{i \neq j} \frac{\cos\theta\, S_i^z S_j^z + \sin\theta\,(S_i^x S_j^x + S_i^y S_j^y)}{|\mathbf{R}_i - \mathbf{R}_j|^3}$$

$S^+ = a^\dagger\sqrt{1-n}$
$S^- = \sqrt{1-n}\,a$
$S^z = a^\dagger a - 1/2$

hard-core boson mapping via Holstein-Primakoff transformation (easiest case: Ising Ferromagnet)

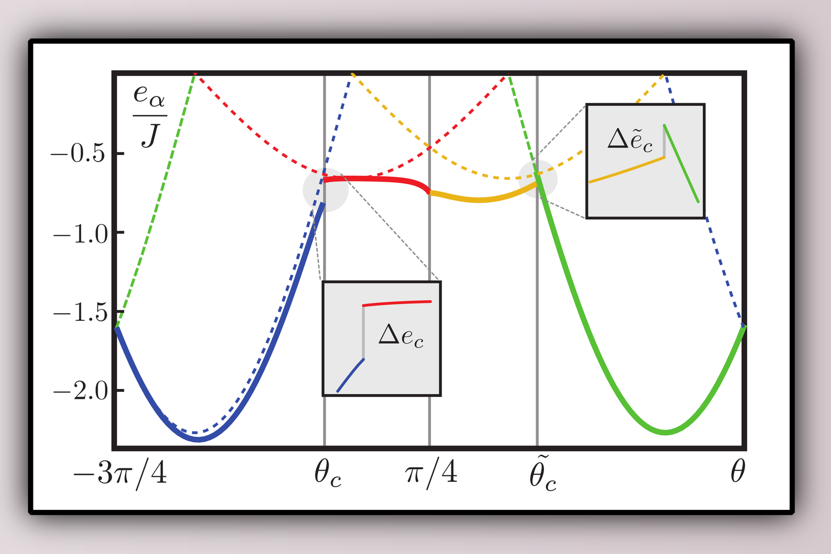
$$H = Ja^3 \sum_{i \neq j} \frac{\cos\theta\,(n_i n_j - n_i + 1/4) + \frac{1}{2}\sin\theta\,(a_i^\dagger a_j + a_j^\dagger a_i)}{|\mathbf{R}_i - \mathbf{R}_j|^3}$$

spin-wave approximation $\langle n_i \rangle \ll 1$ neglect interactions between excitations: $n_i n_j \approx 0$

„long-range tunneling"

Spin-wave excitation spectrum by Fourier (and Bogoliubov) transformation

see part below

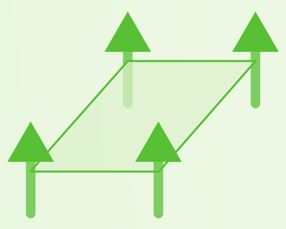


modifications of ground state energy per particle due to spin-wave theory (dashed: mean-field prediction)

jumps at critical angles θ_c and $\tilde{\theta}_c$ could indicate intermediate phases

▷ Peter, Müller, Wessel, Büchler, Phys. Rev. Lett. **109**, 025303 (2012)
▷ Gorshkov et al, Phys. Rev. Lett. **107**, 115301 (2011)
▷ Gorshkov et al, Phys. Rev. A **84**, 033619 (2011)

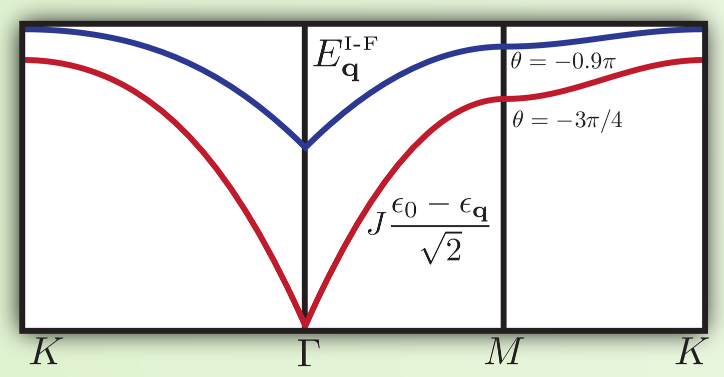
Ising ferromagnetic phase



▷ long-range order at low T

▷ gapped, **linear** excitation spectrum $E_{\mathbf{q}}^{\text{I-F}} = E_\Gamma^{\text{I-F}} + \hbar c\,|\mathbf{q}|$ for $q \to 0$

(compare to nearest neighbor spectrum $\sim q^2$ for $q \to 0$)

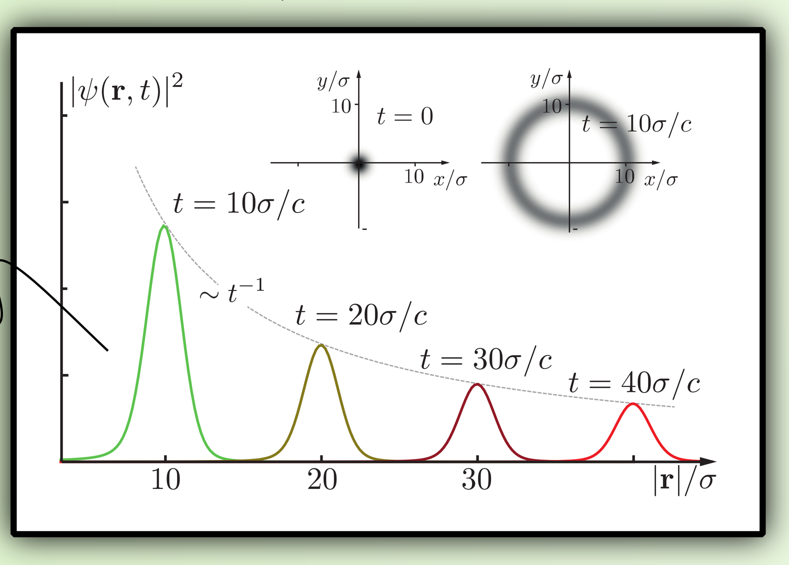


$\Gamma = (0,0)$
$M = (0, \pi/a)$
$K = (\pi/a, \pi/a)$

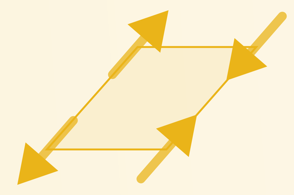
▷ spin-wave dynamics described by ballistic expansion with speed $\boxed{c = -2\pi a J \sin(\theta)}$

(compare to usual spreading of wave-packets)

universal shape of wave-packet



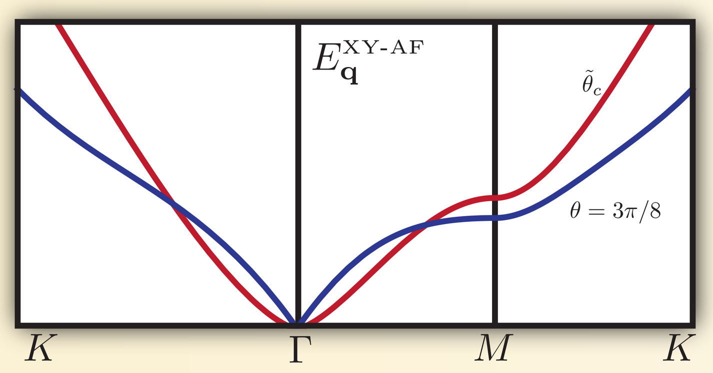
XY-antiferromagnetic phase

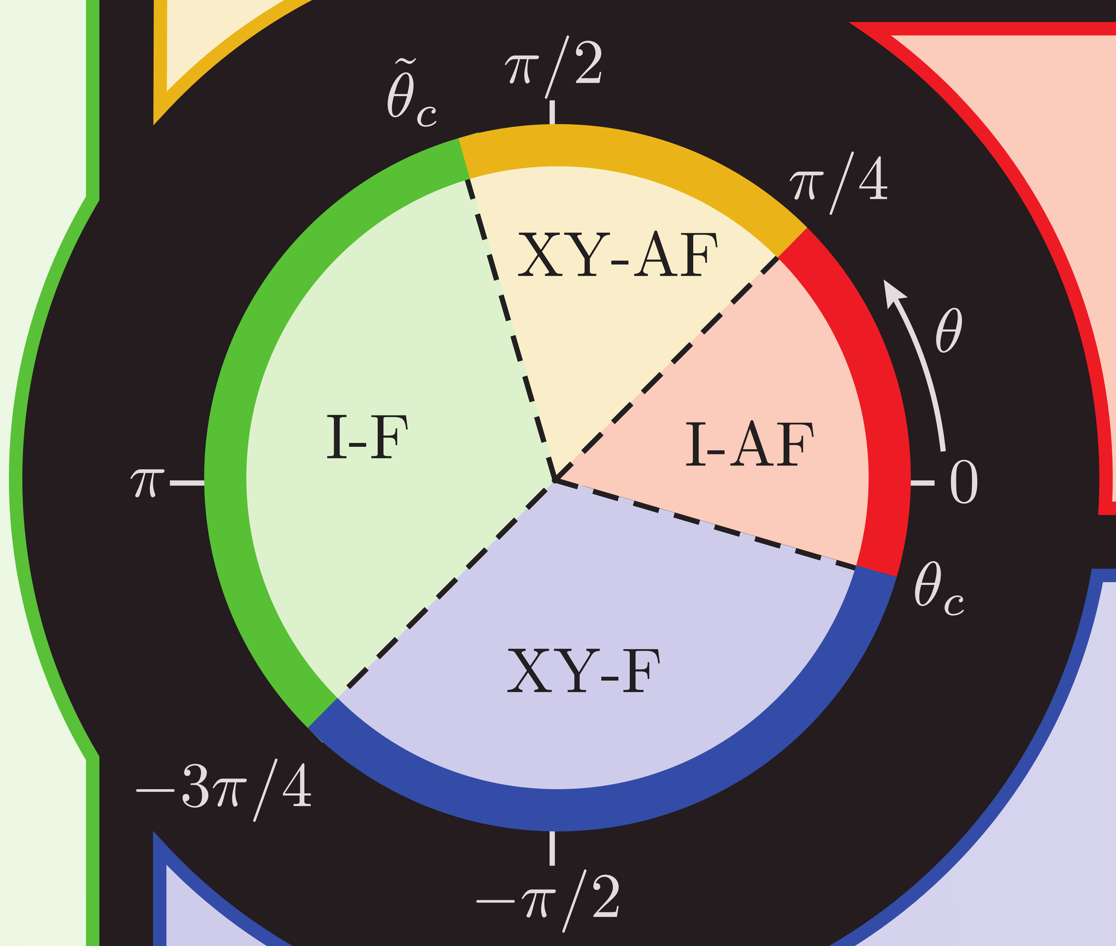


▷ frustration due to second-nearest neighbors

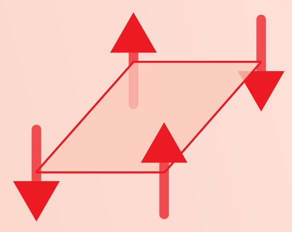
▷ conventional linear Goldstone mode

▷ Kosterlitz-Thouless transition to quasi ordered state





Ising-ferromagnetic phase

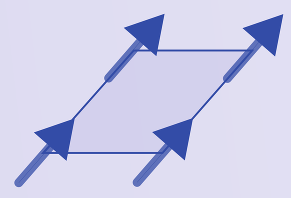


▷ frustration due to second-nearest neighbors

▷ long-range order at low T

▷ algebraic correlations in gapped system

XY-ferromagnetic phase



▷ $\boxed{\text{long-range order at finite T!}}$

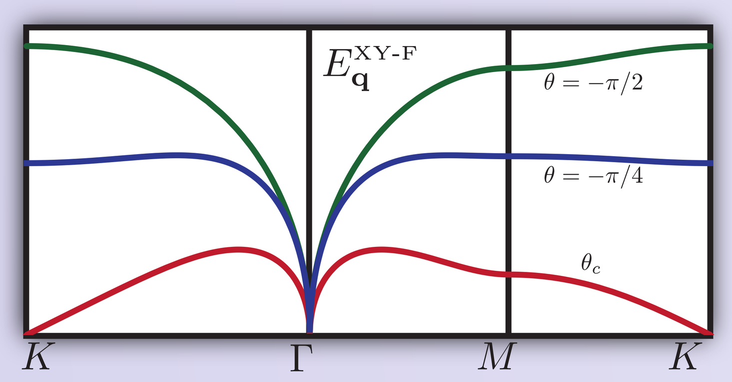
$$\left\langle S_i^x S_j^x + S_i^y S_j^y \right\rangle \to m^2$$

▷ spontaneously broken $U(1)$ symmetry in 2D

▷ Mermin-Wagner requirements not fulfilled

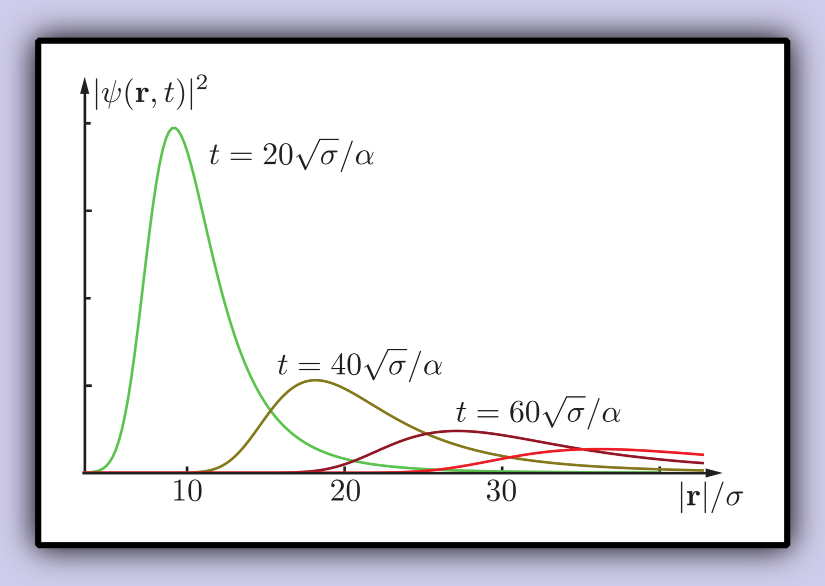
▷ Spin-wave excitation spectrum $E_{\boldsymbol{q}}^{\text{XY-F}} \sim \sqrt{q}$ (for $q \to 0$)

(compare to linear Goldstone mode in nearest-neighbor model and XY-AF phase)



▷ spin-wave dynamics with group velocity $\boxed{v_{\mathrm{q}} \sim 1/\sqrt{q}}$

▷ Gaussian wave-packet of width σ spreads with speed $v \sim \sigma$!



suppression of the order parameter

$$\Delta = \langle S_i^x \rangle - 1/2 = \langle a_i^\dagger a_i \rangle$$

$$= \begin{cases} 0.08 & T = 0 \quad (\theta = -\pi/2) \\ \text{finite} & T > 0 \end{cases}$$

$\to \infty$ for nearest neighbor model

dipolar interaction favors mean-field solution due to additional „neighbors"

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