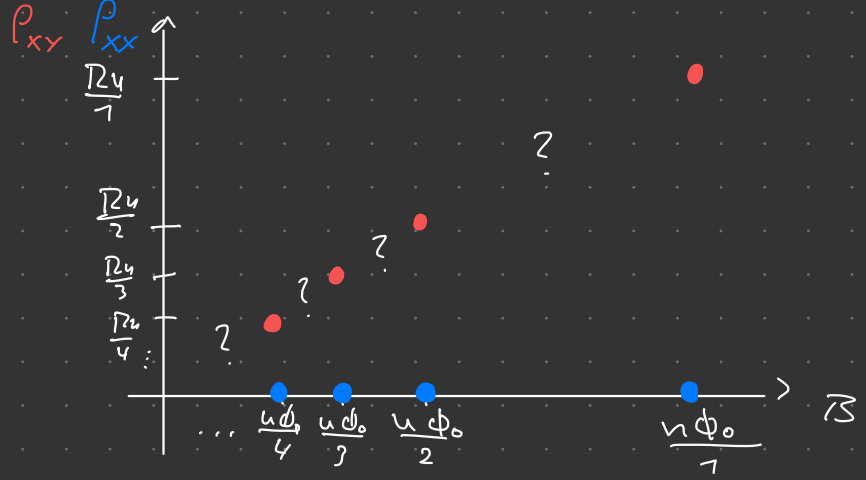
1.1.5. Notes on disorder

1. ✗ System with fixed electron density n

Recall: Number of states per LL: $N = \frac{AB}{\phi_0}$

→ Lowest $\nu \in \mathbb{N}$ LLs exactly filled for: $B_\nu = \frac{\phi_0 n}{\nu}$

→ Only for the discrete B_ν the Hall response σ_{xy} is topological and thus quantized:



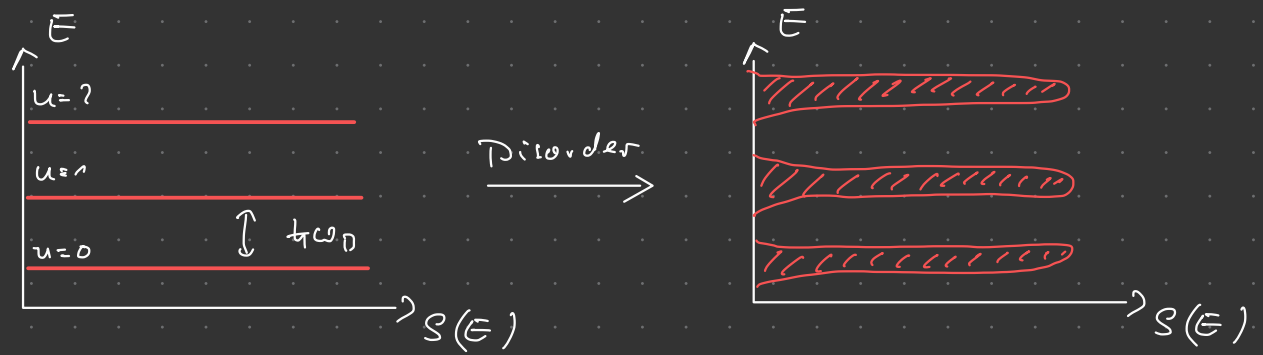
→ This does not explain the observed plateaus!

Solution: Disorder

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2}$$

$$\sigma_{xy} = \frac{-\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}$$

2. First effect of disorder: LL are broadened:



→ This does still not explain the observed plateaus 🙁

3. Second effect of disorder:

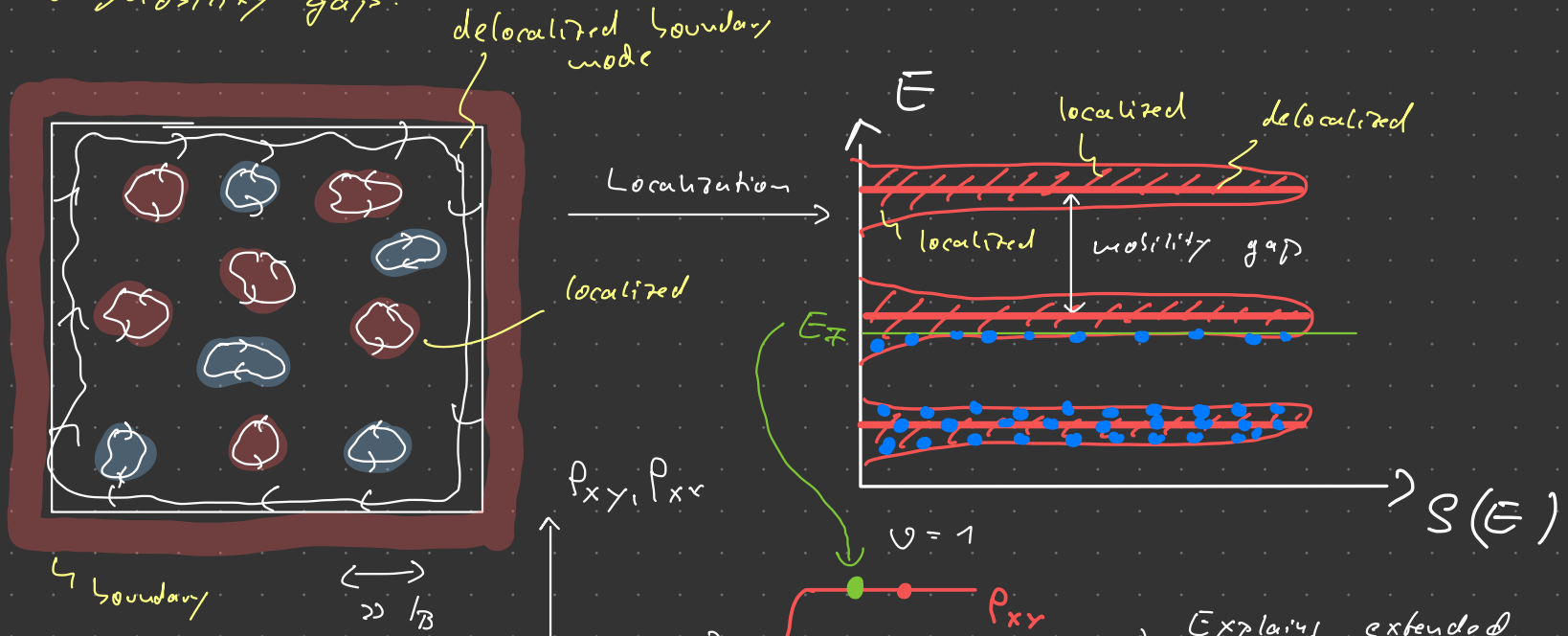
* (Most) single-electron states are localized and pinned at local potential peaks / dips

→ Do not contribute to conductivity

* At least one mode along the edge cannot be localized

→ Contributes to conductivity

→ Mobility gap:



→ Filling/depletion of broadened LLs for $B \geq B_0$ does not affect conductivity as long as E_F is in the mobility gap.

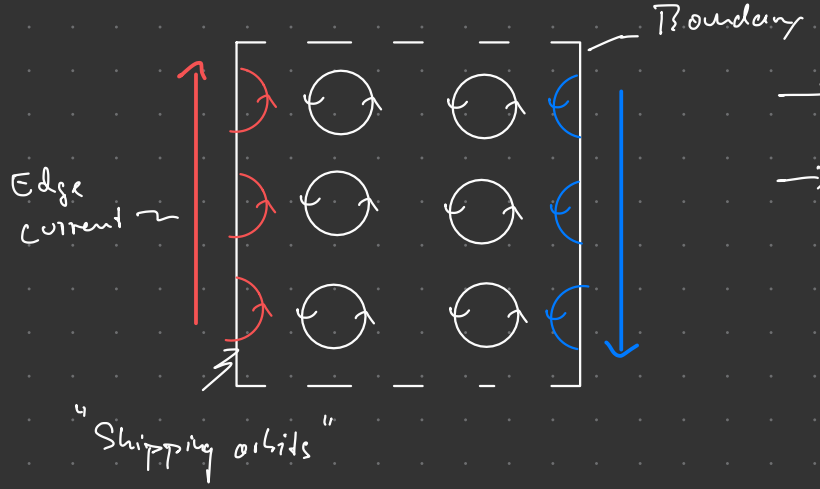
→ Explains extended Hall plateaus around B_0 with quantized height R_K/υ

In a nutshell:

* Topology fixes the height of the plateaus but
* disorder gives them their finite width

1.1.6. Notes on edge states

1. Classical picture:

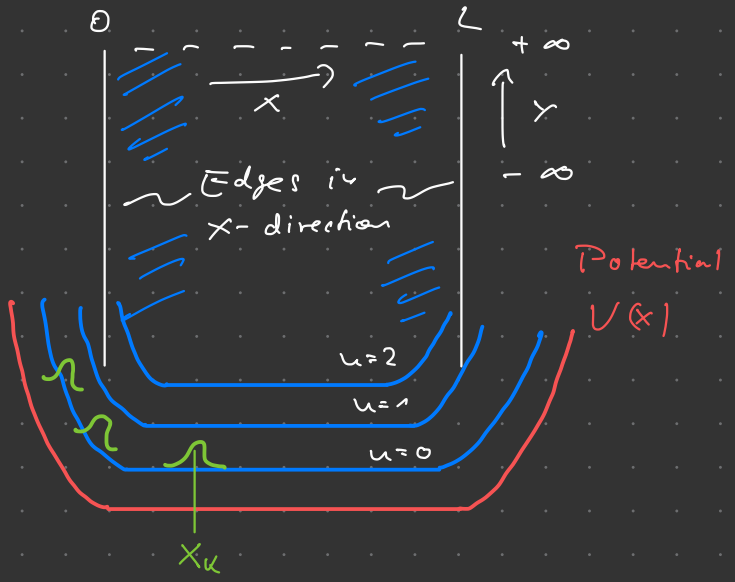


→ Skipping orbits

→ Chiral currents along edges

2. Quantum picture:

a) & Strip geometry:



b) Hamiltonian in Landau gauge:

$$H = \frac{1}{2m} p_x^2 + \frac{m\omega_B^2}{2}\left(x + u l_B^2\right)^2 + V(x)$$

$V(x)$: Potential that varies on length scales $\gg l_B$

c) LL eigenfunctions $\psi_{n,u}$ still eigenfunctions

& LLL: $\psi_{0,u}(x,y) = \mathcal{N} e^{iuy} e^{-\frac{(x+u l_B^2)^2}{2 l_B^2}}$

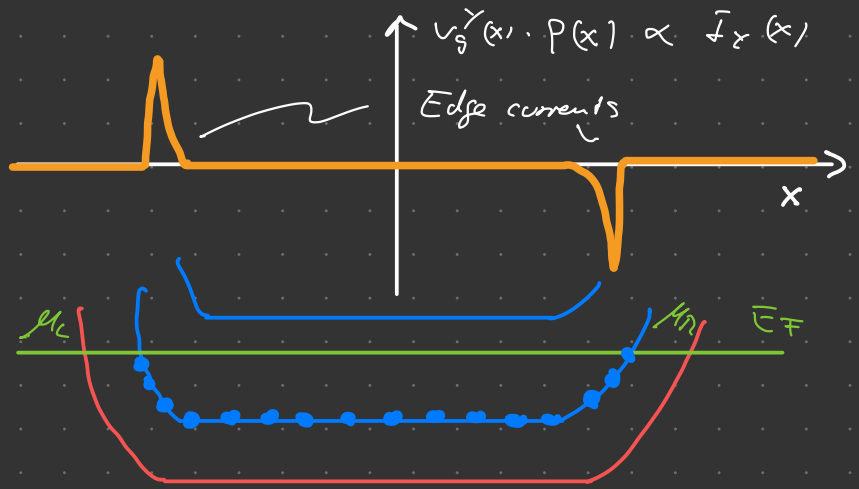
y-momentum $u \rightarrow$ localized at $X_u = -u l_B^2$

$\rightarrow$ Eigenenergy $E_u = \frac{1}{2}\hbar\omega_B + V(X_u)$

d) Group velocity:

$$v_g^y(X) = \frac{1}{\hbar}\frac{\partial E_u}{\partial u} = \frac{1}{\hbar}\frac{\partial E_u}{\partial X_u}\frac{\partial X_u}{\partial u} = -\frac{l_B^2}{\hbar}\frac{\partial V(x)}{\partial x} = -\frac{1}{eB}\frac{\partial V(x)}{\partial x}$$

→ Current density $I_y(x) = -e\, v_g^y(x)\, \rho(x)$



→

$$I_y = \int_{-\infty}^{\infty} I_y(x)\, dx$$

$$= -e \int_{-\infty}^{\infty} \frac{d\mu}{2\pi} \frac{1}{\hbar} \frac{\partial E_\mu}{\partial \mu} \rho(\mu)$$

$$= -\frac{e}{2\pi\hbar} \int_{\mu_L}^{\mu_R} dE$$

$$= -\frac{e}{2\pi\hbar} \underbrace{(\mu_R - \mu_L)}_{=0}$$

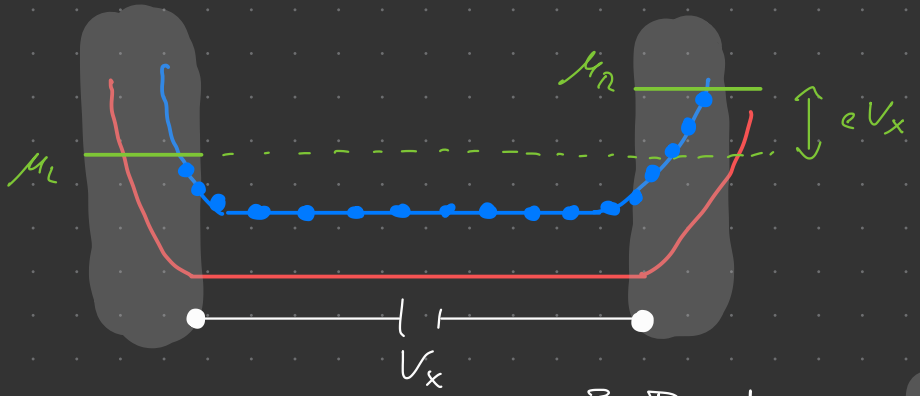
$$= 0$$

e) Hall conductivity:

Apply electric field in x-direction: $\mu_R - \mu_L = e V_x \stackrel{!}{=} V_H$

→ Hall current:

$$I_y = -\frac{e}{2\pi\hbar}(\mu_R - \mu_L) = -\frac{e^2}{2\pi\hbar} V_x$$

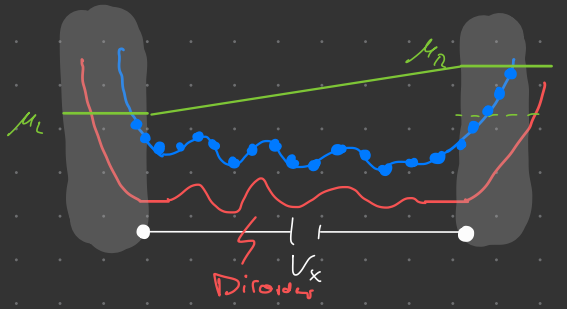


→ Hall conductivity:

$$\sigma_{xy} = -\frac{e^2}{2\pi\hbar} \cdot 1$$

3. Disorder:

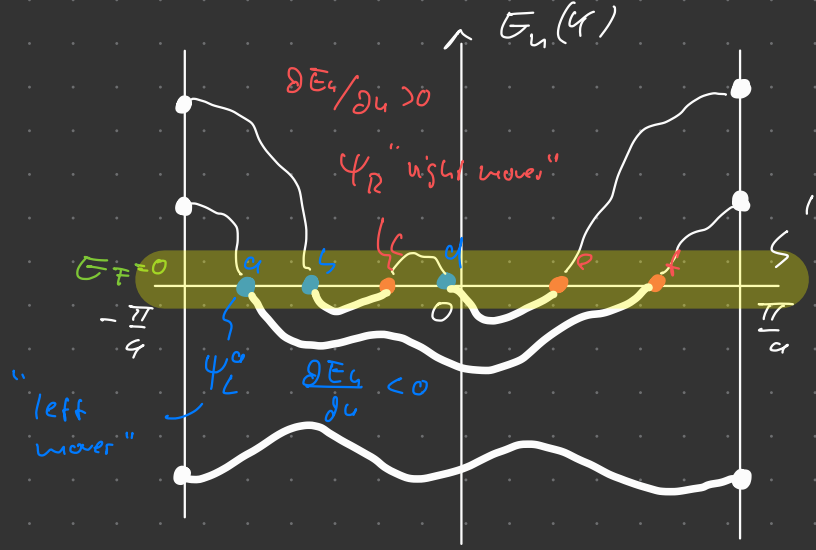
→ The Hall conductivity is robust to disorder ☺



4. Chiral edge modes are special:

Note: Nielsen-Ninomiya-Theorem:

4 Non-interacting fermions on a lattice in 1D:



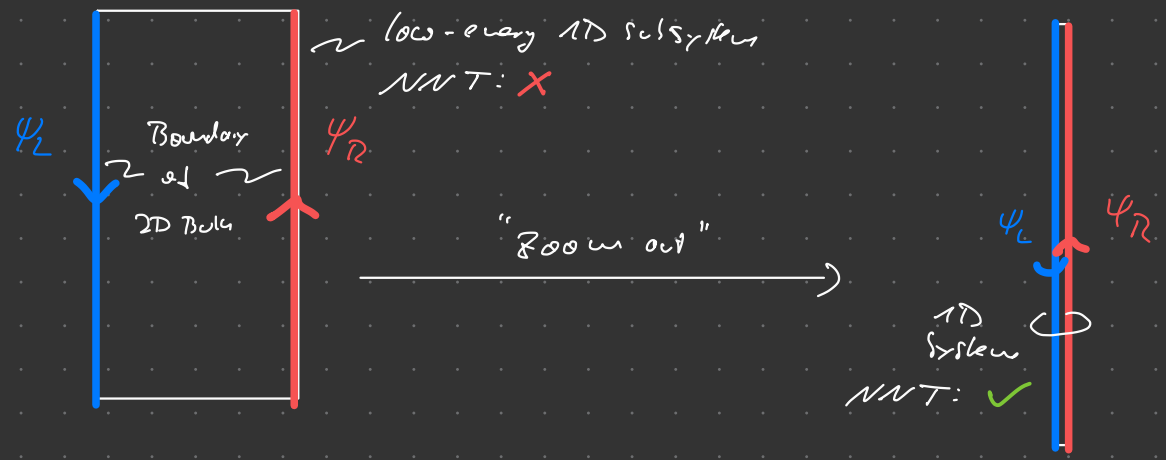
→ Brillouin zone = Circle S^1

low-energy physics

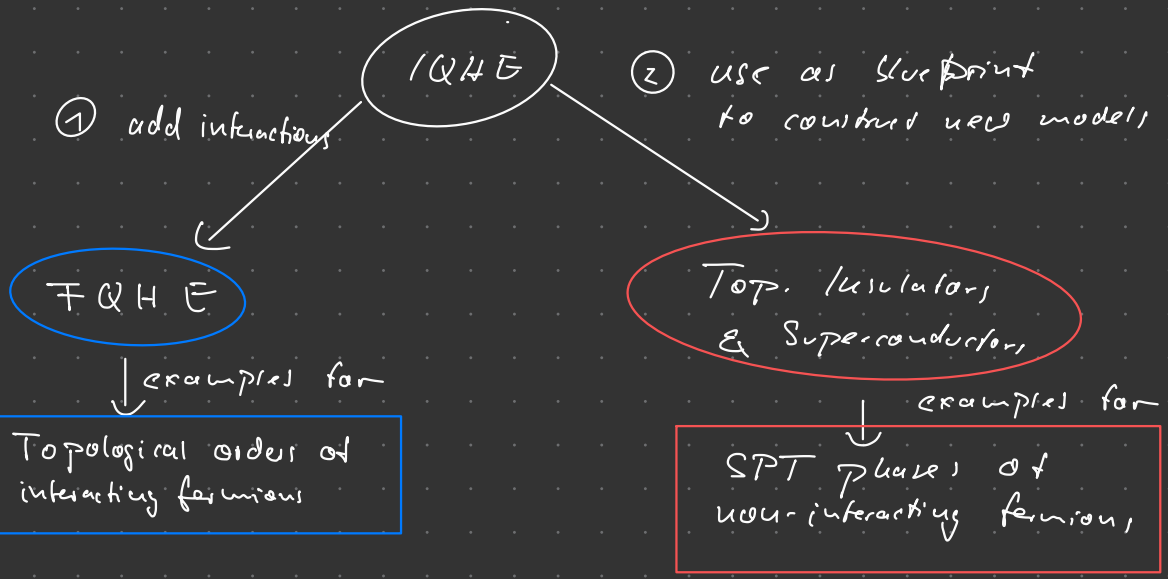
→ Equal number of left (Ψ_L) and right movers (Ψ_R) in low-energy theory of lattice models

→ Chiral edge modes can only appear on the boundary of a 2D bulk material

The magnetic field spatially separates left- and right movers:



1.1.7 Notes on classification



① Keep the QHE setting but consider fractionally filled LLs:

→ Fractional quantum Hall effect (FQHE)

→ Different *topologically ordered* states

Top. Insulators & Superconductors

② Leave QHE setting but stay with non-interacting fermions

→ Construct lattice models with top. Chern bands

- * ... w/o magnetic field
- * ... w/o breaking time-reversal sym.
- * ... w/o particle-number conservation