# Theoretische Physik IV: Statistische Mechanik, Exercise 1

---

*Prof. Dr. Hans Peter Büchler WS 2013/14, 16. October 2013*

Information regarding the lecture and the online version of the exercise sheets, can be found on the webpage **http://www.itp3.uni-stuttgart.de**. The exercise problems are split into two types: **Written** and **Oral**. The **written** problems have to be turned in during the exercise groups, will be corrected by the individual tutor and handed out in the following exercise session. The **oral** exercise problems have to be prepared for the exercise session. The standard procedure is, that at the beginning of the session the prepared problems can be checked on the corresponding list. Afterwards one student is chosen to present the checked problem on the blackboard. The requirement for obtaining the exercise certificate is: turn in the **written** problems and achieve 80% of the available points, prepare and check 66% of the **oral** problems, and present a problem twice on the blackboard during the exercise period.

**1. Partial Derivative (Written)**
The variables $x$, $y$ and $z$ are connected through $f(x, y, z) = 0$. There is a function $w(x, y)$ with two out of the three variables. Show that

$$a)\quad \left.\frac{\partial x}{\partial y}\right|_z = \left(\left.\frac{\partial y}{\partial x}\right|_z\right)^{-1},$$

$$b)\quad -1 = \left.\frac{\partial x}{\partial y}\right|_z \left.\frac{\partial y}{\partial z}\right|_x \left.\frac{\partial z}{\partial x}\right|_y,$$

$$c)\quad \left.\frac{\partial x}{\partial w}\right|_z = \left.\frac{\partial x}{\partial y}\right|_z \left.\frac{\partial y}{\partial w}\right|_z,$$

$$d)\quad \left.\frac{\partial x}{\partial z}\right|_w = \left.\frac{\partial x}{\partial y}\right|_w \left.\frac{\partial y}{\partial z}\right|_w,$$

$$e)\quad \left.\frac{\partial x}{\partial y}\right|_z = \left.\frac{\partial x}{\partial y}\right|_w + \left.\frac{\partial x}{\partial w}\right|_y \left.\frac{\partial w}{\partial y}\right|_z.$$

**2. State variables (Written)**
It is known, that through small deviations $dx$, $dy$ of the external parameters $x$, $y$ the quantity $E$ changes as

$$\delta E = F_x\, dx + F_y\, dy\,,$$

with the vector $\mathbf{F}(x, y) = [F_x(x, y)\,, F_y(x, y)]$. The quantity $E$ is called a state variable, if $\delta E$ is represented through an exact differential

$$dE = \partial_x\, E(x, y)\; dx + \partial_y\, E(x, y)\; dy\,.$$

(a) Consider $\delta E$ (i.e. $\mathbf{F}$), show the equivalence of the following two statements

i. $E$ is a state variable, i.e. $\exists E(x, y):\ \mathbf{F} = \nabla E$ and

ii. $\nabla \wedge \mathbf{F} = 0$ .

(b) Why is $E$ called a state variable?

(c) Why are state variables important in thermodynamics?

(d) If a differential $\delta E$ is not exact, it is possible to find an integrating factor $\mu(x, y)$, such that $dS = \mu(x, y)\,\delta E$ becomes exact. Determine the integrating factor $\mu(x, y)$ for

$$\delta E = \left(xy^2 + xye^x\right) dx + \left(2x^2y + xe^x\right) dy\,,$$

under the assumption, that $\mu$ only depends on $x$. In addition determine $S(x, y)$.

(e) Give an example for an exact differential, a non-exact differential and its integrating factor.

**3. States of equilibrium (Oral)**

A hollow cylinder is separated into two chambers through a piston, see figure 1. The walls of the hollow cylinder shall be isolating. In the initial state, the piston is fixed and also isolating, whereas the chambers (1) and (2) have the volumes $V_1$ and $V_2$ and contain the number of molecules $N_1$ and $N_2$ of the same gas (e.g. helium), respectively. The pressure in this state is $P_1$ and $P_2$, respectively.

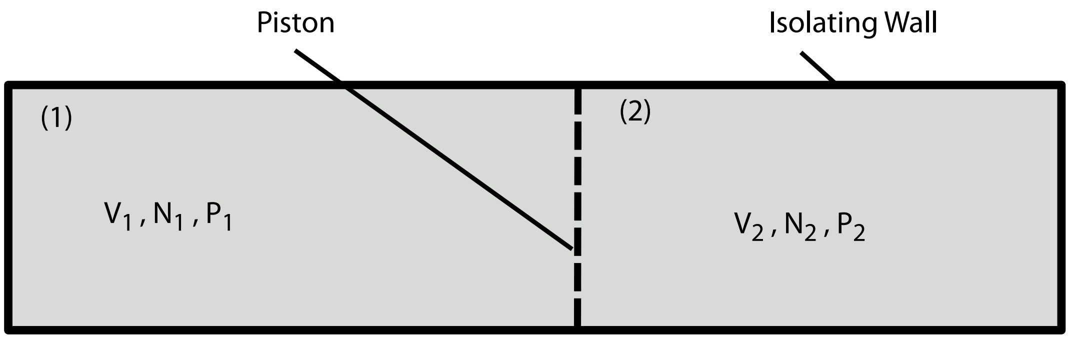


Figure 1: Hollow cylinder, separated into two chambers through a piston.

a) Now the piston shall be diathermic. What is the new state of equilibrium?

b) What is the new state of equilibrium, if the diathermic piston is able to move freely?

c) Does the state of equilibrium of problem b) change, if, in addition, the piston becomes permeable for the molecules of the gas?