

# Theoretische Physik IV: Statistische Mechanik, Exercise 2

---

Prof. Dr. Hans Peter Büchler WS 2013/14, 23. October 2013

## 1. Internal Energy (Written)

Given are two systems  $\Sigma_1$  and  $\Sigma_2$ , see figure 1 with the equations of state

$$\Sigma_1 : U_1 = \frac{3}{2}n_1RT_1 \qquad \Sigma_2 : U_2 = \frac{5}{2}n_2RT_2,$$

which are in diathermic contact. Here  $n$  is the number of moles,  $R$  is the gas constant with  $R = 8.314 \text{ J/K} \cdot \text{mol}$ .

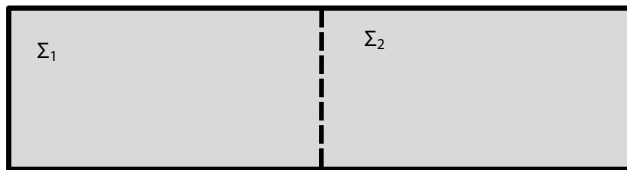


Figure 1: Two systems in diathermic contact.

- The total energy of the combined system shall be  $U$ . Determine the internal energies  $U_1$  and  $U_2$  for  $\Sigma_1$  and  $\Sigma_2$ , respectively, for thermal equilibrium. As an example, insert the values  $n_1 = 2 \text{ mol}$ ,  $n_2 = 3 \text{ mol}$  and  $U = 24000 \text{ J}$ .
- Instead of the total energy, now the initial temperatures  $T_1$  and  $T_2$  of  $\Sigma_1$  and  $\Sigma_2$ , respectively, are given. Determine  $U_1$ ,  $U_2$  and  $T$  for thermal equilibrium, with  $T_1 = 250 \text{ K}$ ,  $T_2 = 300 \text{ K}$ ,  $n_1 = 2 \text{ mol}$  and  $n_2 = 3 \text{ mol}$ .

## 2. Ideal Gas, Part 1 (Written)

- Show, for the case of an ideal gas, that the relation

$$\frac{\partial^2 U}{\partial S \partial V} = \frac{\partial^2 U}{\partial V \partial S},$$

holds.

- From the lecture we know the following relations

$$\begin{aligned} dU &= \delta Q - \delta W, \\ \delta Q &= C_V dT, \\ \delta W &= p dV. \end{aligned}$$

Hence

$$\left. \frac{\partial U}{\partial V} \right|_T = -p.$$

This is obviously wrong for the ideal gas, where  $U = U(T)$ , i.e.  $U$  is independent of  $V$ . Where is the mistake?

### 3. Ideal Gas, Part 2 (Oral)

- The ideal gas law is  $PV = nRT$ , where  $n$  again is the number of moles of gas and  $R$  the gas constant. A process equation is a simple function of, usually, two variables of  $P, V, T$ , which remains constant throughout the process. Find isobaric, isothermic and isochoric process equations.
- Apply the ideal gas law to get the adiabatic process equation  $PV^\gamma = const.$ , where  $\gamma = C_P/C_V$ .
- What is the meaning of a reversible process? Explain how to perform an adiabatic process in an reversible procedure.
- Consider the free expansion of a gas, figure 2. This is also an adiabatic expansion, but of irreversible nature. Why?
- During a free expansion,  $\delta Q$ ,  $dU$  and  $\delta W$  are zero, respectively, but still the entropy increases. Find the expression for the increasing entropy.

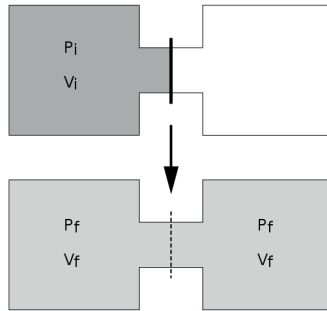


Figure 2: Upper part: Two chambers, separated by a valve with initial pressure  $P_i$  and initial volume  $V_i$ . Lower part: System after opening the valve with final pressure  $P_f$  and final volume  $V_f$ .

### 4. Equation of State for magnetic Substances (Oral)

A homogeneous magnetic field  $\mathbf{H}$  is created by a long coil. Then, an isotropic, magnetic material shall be placed in the center of the coil. The reversible work performed by the coil onto the material is given by

$$\delta A = H dM,$$

at unity volume. Here  $M$  is the magnetization of the material. Since we are dealing with an isotropic material, the vector nature of  $H$  and  $M$  can be neglected.

- Write down the entropy of the system as  $S = S(T, H)$  and derive with this the relationship between the magnetization  $M = M(T, H)$  (thermic equation of state) and the internal energy  $U = U(T, H)$  (caloric equation of state).
- A paramagnetic substance fulfills the Curie-law

$$M = K \cdot \frac{H}{T},$$

with  $K$  being a material dependent constant. Show that  $U$  only depends on  $T$ .