1. Internal Energy (Written)

Given are two systems $\Sigma_1$ and $\Sigma_2$, see figure 1 with the equations of state

\[ \Sigma_1 : U_1 = \frac{3}{2} n_1 RT_1 \]
\[ \Sigma_2 : U_2 = \frac{5}{2} n_2 RT_2 , \]

which are in diathermic contact. Here $n$ is the number of moles, $R$ is the gas constant with $R = 8.314 \text{ J/K} \cdot \text{mol}$.

(a) The total energy of the combined system shall be $U$. Determine the internal energies $U_1$ and $U_2$ for $\Sigma_1$ and $\Sigma_2$, respectively, for thermal equilibrium. As an example, insert the values $n_1 = 2 \text{ mol}$, $n_2 = 3 \text{ mol}$ and $U = 24000 \text{ J}$.

(b) Instead of the total energy, now the initial temperatures $T_1$ and $T_2$ of $\Sigma_1$ and $\Sigma_2$, respectively, are given. Determine $U_1$, $U_2$ and $T$ for thermal equilibrium, with $T_1 = 250 \text{ K}$, $T_2 = 300 \text{ K}$, $n_1 = 2 \text{ mol}$ and $n_2 = 3 \text{ mol}$.

2. Ideal Gas, Part 1 (Written)

(a) Show, for the case of an ideal gas, that the relation

\[ \frac{\partial^2 U}{\partial S \partial V} = \frac{\partial^2 U}{\partial V \partial S} , \]

holds.

(b) From the lecture we know the following relations

\[ dU = \delta Q - \delta W , \]
\[ \delta Q = C_V dT , \]
\[ \delta W = p dV . \]

Hence

\[ \frac{\partial U}{\partial V} \bigg|_T = -p . \]

This is obviously wrong for the ideal gas, where $U = U(T)$, i.e. $U$ is independent of $V$. Where is the mistake?
3. Ideal Gas, Part 2 (Oral)

(a) The ideal gas law is \( PV = nRT \), where \( n \) again is the number of moles of gas and \( R \) the gas constant. A process equation is a simple function of, usually, two variables of \( P, V, T \), which remains constant throughout the process. Find isobaric, isothermic and isochoric process equations.

(b) Apply the ideal gas law to get the adiabatic process equation \( PV^\gamma = \text{const.} \), where \( \gamma = C_P/C_V \).

(c) What is the meaning of a reversible process? Explain how to perform an adiabatic process in an irreversible procedure.

(d) Consider the free expansion of a gas, figure 2. This is also an adiabatic expansion, but of irreversible nature. Why?

(e) During a free expansion, \( \delta Q, dU \) and \( \delta W \) are zero, respectively, but still the entropy increases. Find the expression for the increasing entropy.

\[ \text{Figure 2: Upper part: Two chambers, separated by a valve with initial pressure } P_i \text{ and initial volume } V_i. \text{ Lower part: System after opening the valve with final pressure } P_f \text{ and final volume } V_f. \]

4. Equation of State for magnetic Substances (Oral)

A homogeneous magnetic field \( H \) is created by a long coil. Then, an isotropic, magnetic material shall be placed in the center of the coil. The reversible work performed by the coil onto the material is given by

\[ \delta A = H \, dM, \]

at unity volume. Here \( M \) is the magnetization of the material. Since we are dealing with an isotropic material, the vector nature of \( H \) and \( M \) can be neglected.

(a) Write down the entropy of the system as \( S = S(T, H) \) and derive with this the relationship between the magnetization \( M = M(T, H) \) (thermic equation of state) and the internal energy \( U = U(T, H) \) (caloric equation of state).

(b) A paramagnetic substance fulfills the Curie-law

\[ M = K \cdot \frac{H}{T}, \]

with \( K \) being a material dependent constant. Show that \( U \) only depends on \( T \).