Theoretische Physik IV: Statistische Mechanik, Exercise 3

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1. **Carnot Cycle with ideal Gas (Written)**
   Show, that the efficiency of a Carnot machine is

$$\eta_{\text{id. gas}} = 1 - \frac{T_2^{\text{gas}}}{T_1^{\text{gas}}} ,$$

   when an ideal gas is used as medium.
   For this, calculate the work and heat for each step of the cycle.

2. **Otto Cycle (Oral)**
   Convert the given sketch/figure (1) into a T - S diagram. Calculate the efficiency

$$\eta_{\text{Otto}} := \frac{\text{performed work}}{\text{delivered heat}} ,$$

   of this machine, for an ideal gas as medium with $c_{\text{V}} = \text{const}$. In addition show

$$\eta_{\text{Otto}} \; < \; \eta_{\text{Carnot}} = 1 - \frac{T_{\text{min}}}{T_{\text{max}}} .$$

   Hint
   The equation for the difference in entropy from S_0 to S is

$$S - S_0 = \int_{T_0}^{T} c_V \frac{dT}{T} + \int_{V_0}^{V} R \frac{dV}{V} .$$

   For an adiabatic process, the equation relating T and V is

$$T V^{R/c_V} = T_0 V_0^{R/c_V} .$$

3. **Water Carnot Cycle (Oral)**
   Here we consider water as a medium in a Carnot cycle. The coefficient of expansion is given by

$$\alpha = \frac{1}{V} \frac{\partial V}{\partial T}\bigg|_p ,$$

   and has the following properties, depending on the temperature of water T_w,

   - if $T_w > 4°\text{C}$, then $\alpha > 0$,
   - if $T_w = 4°\text{C}$, then $\alpha = 0$,
   - if $T_w < 4°\text{C}$, then $\alpha < 0$.

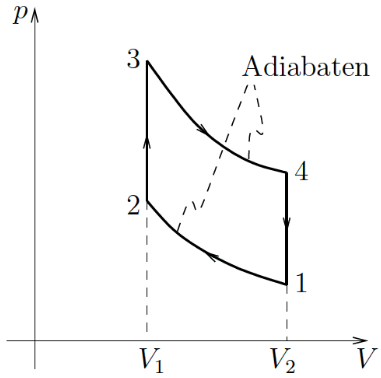

Figure 1: p - V diagram for the Otto cycle.

(a) Consider two water isotherms at 6°C and 2°C. How must the volume change, such that heat is always supplied?
Hint
The isothermic compressibility $\kappa_T = -V^{-1} \left(\partial V / \partial p \right)_T$ is always positive, because applying more pressure to a liquid does not make it larger. The Helmholtz free energy is given by $dF = -S\, dT - p\, dV$, use the fact of its exactness to derive a relation between S and p.

(b) Why is it not possible to build a Carnot cycle with the isotherms at the two temperatures given in (a)?
Hint
Consider the process in the T - V diagram and calculate the slope $\left(\partial T / \partial V \right)_S$ of both adiabates. The heat capacity at constant volume is given through $c_V = \left(\partial U / \partial T \right)_V$.

4. Magnetic Carnot Machine (Oral)

Consider a paramagnetic material as in exercise sheet 2, problem 4, which has a constant heat capacity $c_M = \left(\partial U / \partial T \right)_M$. The material shall be used as a Carnot machine between two heat reservoirs of temperatures $T_2 > T_1$.

(a) Find the equation of the adiabates of the system and sketch a cycle for the machine in a T - M diagram. In which direction has the cycle to run, in order to perform work?

(b) Calculate the work performed by the machine in one cycle.

(c) Compute the efficiency of the machine.