Theoretische Physik IV: Statistische Mechanik, Exercise 4

Prof. Dr. Hans Peter Büchler WS 2013/14, 5. November 2013

1. Entropy differential (Oral)

Show that

$$TdS = \left(\frac{\partial U}{\partial T}\Big|_p + p\frac{\partial V}{\partial T}\Big|_p \right) dT - T\frac{\partial V}{\partial T}\Big|_p dp. \tag{1}$$

<u>Hint</u>

(a) Use exactness of $S = S(T, p)$.
(b) Understand the derivation of the analogous relation

$$TdS = \frac{\partial U}{\partial T}\Big|_V dT + T\frac{\partial p}{\partial T}\Big|_V dV \tag{2}$$

from the script "Theorie der Wärme" by Gianni Blatter.

2. Ideal Gas (Written)

Knowing the thermal expansion coefficient α

$$\alpha = \frac{1}{V}\frac{\partial V}{\partial T}\Big|_P, \tag{3}$$

and also isothermal and adiabatic compressibility factors (κ_T and κ_S)

$$\kappa_T = -\frac{1}{V}\frac{\partial V}{\partial P}\Big|_T \text{ and } \kappa_S = -\frac{1}{V}\frac{\partial V}{\partial P}\Big|_S, \tag{4}$$

show following relations for heat capacity at constant volume c_V and heat capacity at constant pressure c_P

(a)

$$c_V = \frac{TV\alpha^2\kappa_S}{(\kappa_T - \kappa_S)\,\kappa_T}, \tag{5}$$

(b)

$$c_P = \frac{TV\alpha^2}{\kappa_T - \kappa_S}. \tag{6}$$

<u>Hint</u>

(a) Use Eq. (2) and Eq. (1) as a starting point.

3. Thermal equilibrium and entropy (Oral)

There are given two objects of the same type (same material and volume) but at different temperatures ($T_1 > T_2$). We place them in thermal contact (see Fig.1) and let them achieve the state of thermal equilibrium.

(a) Calculate the temperature T_m of two objects at equilibrium.
(b) Calculate the change of the total entropy ΔS.
(c) Show that always $\Delta S > 0$.
(d) Do we deal here with irreversible or reversible process? Justify.

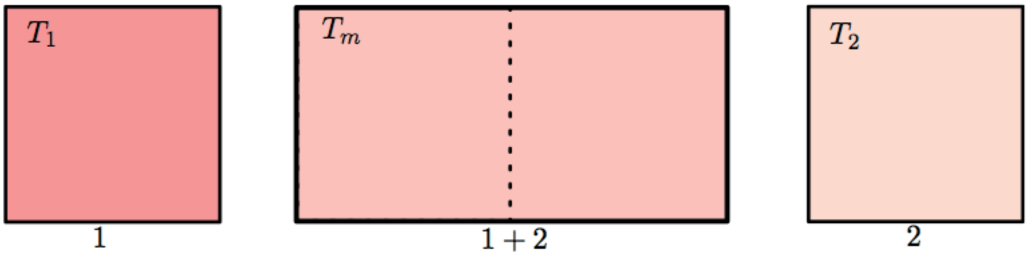

Figure 1: Temperature equilibration ($T_1, T_2 \to T_m$) of two objects in thermal contact.

Hint

(a) There is no heat loss during thermal equilibration. Thermal energy is $Q = mcT$ with mass m, specific heat coefficient c and temperature T.