Theoretische Physik IV: Statistische Mechanik, Exercise 6

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1. Absolute Zero (Oral)

Here we show that absolute zero cannot be reached by an adiabatic expansion. From the lecture we know that

$$c_p \to 0 \qquad \text{for} \quad T \to 0. \tag{1}$$

Thus the specific heat at constant pressure takes on the form

$$c_p = T^x(a_0 + a_1 T + a_2 T^2 + \ldots) \tag{2}$$

where x is a positive exponent.

(a) Show that

$$\frac{V\alpha}{c_p} = \text{const} \neq 0 \qquad\qquad \text{for} \quad T \to 0. \tag{3}$$

(b) Using the result from (a) show that absolute zero cannot be reached by an adiabatic expansion.

In the following we gain intuition whether absolute zero can be reached at all. We consider the fact that cooling processes always take place between two curves with $X = \text{const.}$, e.g. $X_1 = P_1, X_2 = P_2$ $(P_1 > P_2)$.

(c) Draw the $T - S$ diagram and show that absolute zero could only be reached by infinitely many steps. To this end, consider steps consisting of adiabatic cooling and isothermical compression between two isobars. Is it possible to decrease the entropy better than by doing it at $T = \text{const}$?

(d) Imagine a substance described by the $T - S$ diagram as shown in Figure 1. This substance would reach $T = 0$ after a finite number of adiabatic coolings and isothermal compressions. Why such a substance cannot exist?

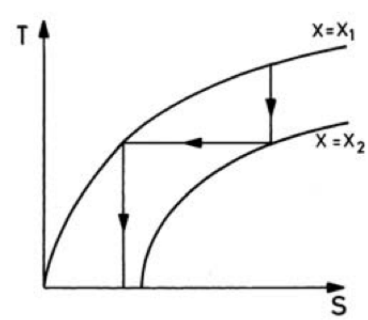
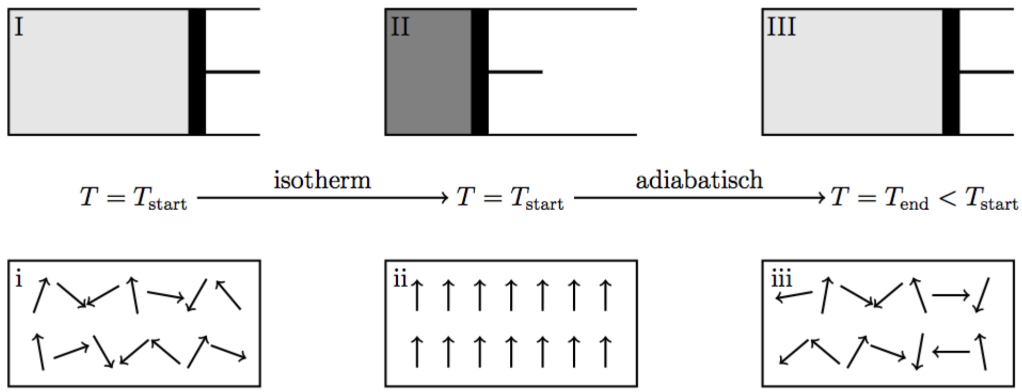


Figure 1: Hypothetical diagram.

2. Adiabatic Demagnetization: The Way to Millikelvin Temperatures (Written)

In adiabatic processes the temperature of a system changes; for example an expanding (ideal) gas cools down. The analogous phenomenon for magnetic substances is called the *magnetocaloric effect*: a paramagnet cools down during adiabatic demagnetization. In this way temperatures of a few millikelvin can be reached.



Assumptions:

As in previous exercises, the work is given by $\delta W = -HdM$ and the Curie law can be formulated as $M(T, H) = KH/T$. The specific heat at $H = 0$ is given by

$$c_H(T, H = 0) = T \frac{\partial S}{\partial T}\bigg|_H = b/T^2, \tag{4}$$

with the positive, material-dependent constants $K, b > 0$.

(a) **The Gibbs potential**: Derive, using the internal energy $U(S, M)$, the Gibbs potential for the paramagnetic substance $G(T, H)$. Next, calculate its differential dG. Which Maxwell relation can be derived from G?

(b) **Adiabatic equation**: Use the result from (a) to calculate the entropy of the system $S(T, H)$.
Hint:
Using the expression for $c_H(T, H = 0)$ (see above) one can determine the 'integration constant'.
How does change the temperature T with the change of the magnetic field H for the constant entropy S?

(c) **Experimental procedure**: Sketch the process for the ideal gas (I $\rightarrow$ II $\rightarrow$ III) in the $T - V$ diagram, and respectively for the paramagnet (i$\rightarrow$ ii $\rightarrow$ iii) in the $T - H$ diagram. To do this, use the entropy relations for the corresponding systems.

(d) **Lowest temperature record**: How must the experimental parameters T, H, and the material properties K and b be chosen, in order to achieve the lowest possible temperature? What are the limiting factors?