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## Problem 1: Stark effect for a harmonic oscillator (Written)

**Learning objective**

The goal of this assignment is to apply a static perturbation (as derived in the lecture) up to second order for a simple setup. This setup has the special property that an exact solution exists and therefore the perturbation theory can be tested by a comparison with the exact solution.

The Hamiltonian of a one-dimensional harmonic oscillator within a homogeneous electric field $E$ is given by

$$\mathrm{H} = \frac{1}{2}\left(P^2 + Q^2\right) + eEQ\,, \tag{1}$$

where the Hamiltonian is dimensionless. Consider the second term of the Hamiltonian as a perturbation to the free oscillator scenario, such that $\mathrm{H}_1 = Q$, with $\lambda = eE$.

a) Calculate the perturbed eigenfunctions and energy eigenvalues up to second order in $\lambda$.

b) Compare the result of perturbation theory with the exact solution for the energy of the problem.

## Problem 2: Fine structure of the hydrogen atom (Oral)

**Learning objective**

Here, we can apply the previously studied perturbation theory to derive the leading order energy corrections to the hydrogen atom due to spin-orbit coupling. This problem combines both topics, perturbation theory and addition of angular momenta, in one task.

Calculate the energy corrections to the $2p$ levels of the hydrogen atom (with the principal quantum number $n = 2$ and orbital angular momentum $\ell = 1$) arising from the spin-orbit coupling within the first order of perturbation theory (See exercise sheet 3).

*Hint:* For the radial part of the matrix elements use $\langle\psi_{2,1,m}|1/r^3|\psi_{2,1,m}\rangle = 1/24a_0^3$, where $\psi_{2,1,m}$ is an eigenfunction of $H_0$, and $a_0 = \hbar^2/m_e e^2$ is the Bohr radius.

## Problem 3: Perturbation in the two level system (Oral)

**Learning objective**

This exercise is another benchmark of the pertubation theory. Here you will practice the special case of a degenerated system and compare it to the exact theory in the end.

The unperturbed Hamiltonian of a two level system in a suitable basis reads

$$H_0 = \begin{pmatrix} E_+ & 0 \\ 0 & E_- \end{pmatrix}$$

Now, take the perturbed Hamiltonian

$$H = H_0 + \lambda n \cdot \sigma,$$

where $n$ is an arbitrary vector of $\mathbb{R}^3$ and $\sigma_i$ a Pauli matrix.

a) Determince the eigenenergies and eigenstates of $H_0$ up to second order for the non-degenerated case ($E_+ \neq E_-$).

b) Now, for a degenerated system ($E_+ = E_-$), find the unperturbed eigenstates which diagonalise the pertubation. What is the first order energy correction?

c) Solve the Schrödinger equation $H\,|\Psi\rangle = E\,|\Psi\rangle$ exactly and compare it to the previous results.